

Innovational Approach to the Mathematics Teaching at the Technical Universities

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Abstract

At the faculties of technical universities, teaching of mathematics should be aimed at the development of students' abilities to apply their knowledge of mathematics to the technology, modelling as well as numerical solutions of engineering problems. That is why, taking into consideration the innovation of curricula of the structured study as well as introduction of a new subjects "Applied Mathematics" to engineering studies at the Faculty of Manufacturing Technologies of the TU in Košice, the goal of mathematicians is to deepen applicative character of mathematical disciplines through the implementation of technical applications to the teaching, to make the teaching process more effective by utilisation of IKT and through the introduction of new teaching methods. The intention of the article is to present the possibilities of MS Excel utilisation with applications of a selected type of differential equations, thus contributing to the above mentioned aim.

1. Introduction

Considering the key thoughts of the Lisbon process, a mathematics teacher at the university of a technical orientation is responsible for a successful or unsuccessful work of technicians in the field of research and development, or in technical practice (Ježek, 2006). Therefore, a common effort of mathematicians at technical universities within a newly created European academic space is to lead students towards realizing the importance of mathematics and its key role in technical subjects, to search for more effective methods and approaches to the study of mathematics, to utilize the mathematical knowledge in their branch of study more purposefully and effectively and to be able to apply the CAS programs (e.g. Matlab, Maple, Mathematica, Statistika 7, etc.) to the solution of complex applicative problems as well as to the mathematical modelling in technical practice (Velichová, 2006). Even a partial fulfillment of these goals could contribute to the flexibility and increase in employment of graduates in a given study branch.

Teaching of mathematics, physics and professional subjects, however, at many faculties of technical universities of the Slovak Republic and the Czech Republic is mutually isolated (Fulier, 2001; Čizmár, 2001). As a result of a reduced time devoted to the teaching of mathematics due to the introduction of a structured study, there is a lack of motivational and applicative tasks as well as the tasks requiring a non-specific transfer: problem-solving and creative tasks. This conclusion was affirmed by the results of the research carried out in 2003 and focused on the evaluation of the current state in mathematics education at the FVT TU in Košice. (Vagaská, 2004). As many as 71.6% of the students included in the research stated that they are able to apply their mathematical knowledge to professional subjects partially and with problems; 7.7% of students admitted they are not able to apply their mathematical

knowledge to professional subjects of technical orientation at all.

Elimination of such problems in mathematics teaching can be reached only through widening and deepening of applicative character of mathematics in specific fields of technology which can be considered as one of important aspects of mathematics teaching modernization. (Fulier,2001; Čizmar,2001). Having in mind the complexity of mathematical calculations in technical applications, the utilization of computing technique and information technology seems to be inevitable and natural in teaching of mathematical applications. Therefore, in accordance with the introduction of the quality management at the FVT, the contents and forms of education are being continuously updated and modernized , new modern elements are being introduced in education.

At present, at the FVT there are on-line study materials created and used in the subjects Numerical Mathematics and Statistics which are available on CDs or at <http://nummet.host.sk>. It is necessary to create such materials also for other mathematics disciplines. The above mentioned opinions support the topicality and importance of the solution of the presented problems.

2. Design of Mathematical Model of the Given Problem in the Form of Differential Equation

In order to increase the motivation level and to eliminate the absence of applicable tasks, we recommend to solve the following task (Fulier, 2001) at practical classes of mathematical analysis or the so called computing practical classes of numerical mathematics:

When blowing through the forest, wind loses its velocity due to the resistance of the trees. It was validated experimentally that the loss of a wind velocity is proportional to the length of this path and the magnitude of a wind velocity. Find the wind velocity heading to inwards the forest at a distance of 150 m from the edge of the forest, if the wind initial velocity at the edge of the forest was $v_0 = 12 \text{ ms}^{-1}$ and at a distance of 10 m from the edge of the forest the wind velocity reduced to the value $v_1 = 9,85 \text{ ms}^{-1}$.

With the application of the theory of differential equations to the above presented problem and with the design of a mathematical model we have to consider that this refers to a proportionally retarded motion and according to the physical laws it holds : $dv = -k \cdot v \, ds$, where $v = v(s)$ is the wind velocity at a distance s from the edge of the forest. A mathematical model of this applicable task is an ordinary differential equation.

$$\frac{dv}{ds} = -k \cdot v \quad (1)$$

with the initial condition $v(0)=12$ and with a supplementary condition $v(10)=9,85$. It is evident that this refers to a separable differential equation, hence we will obtain a universal solution in the form $v = C \cdot e^{-k \cdot s}$. Utilising the initial condition we will determine the value of a arbitrary constant $C = 12$. Using a supplementary condition $v(10)=9,85$ we will determine a constant k from the equation $9,85 = 12 \cdot e^{-10k} \Rightarrow k = \frac{1}{10} \cdot \ln \frac{12}{9,85}$. Taking into consideration

(1) and the constant k , for the given situation we will obtain a differential equation of the wind velocity reduction in the form:

$$\frac{dv}{ds} = \frac{1}{10} \ln \frac{9,85}{12} \cdot v \quad (2)$$

whence by the variables separation and the utilisation of the initial condition we will obtain the solution to the equation:

$$v = 12 \cdot \left(\frac{9,85}{12} \right)^{\frac{s}{10}} \quad (3)$$

It is evident that the same solution can be obtained by the constants k and C substitution for a universal solution. As we are interested in the wind velocity at the distance of 150 m from the edge of the forest, it is enough to substitute $s = 150$ m in the relation (3) and we obtain that the wind velocity at this distance is only $v = 0,62087763 \text{ ms}^{-1}$.

3. Numerical Solution with MS Excel Utilisation

Let us illustrate how a discrete solution to the differential equation (2) with the initial condition $v(0) = 12$ can be found by numerical methods. A numerical solution to the equation (2) will not be obtained in the form of the function $v = v(s)$ which expresses the wind velocity reduction, we will obtain only its approximation in the form of couples $(s_i), (v_i)$ presented in Excel in a column form. For an approximate solution, Euler's method and the 2nd order Runge-Kutta method with the step $h = 10$ will be used. During the problem solving it will follow that the equation (2) requires the solution within the interval in metres $\langle 0, 150 \rangle$ expressing a wind distance from the forest edge. According to the algorithm of Euler's method of the differential equation $v'(s) = f(s, v)$ solution within the interval $\langle s_0, s_0 + n.h \rangle$ with the step h and the initial condition $v(s_0) = v_0$ we write into Excel cells the required relations, see fig.1. In column A we will create a numerical sequence expressing a change of the wind distance from the forest edge starting with the distance of 0 m. In the cell B3, the initial condition is used. In D3 there is a record of the right side of the differential equation (2) in the form $v'(s) = \frac{1}{10} \ln \frac{9,85}{12} \cdot v$, which after being rewritten into Excel looks as follows: $= (1/10) * \text{LN}(9,85/12) * \text{B3}$. Velocity v_1 in the distance $s_1 = 10$ m is expressed according to the algorithm of Euler's method $v_1 = s_0 + h \cdot v'(s_0) = s_0 + h \cdot f(s_0, v_0)$ in the cell B4 by the formula: $= \text{B3} + \text{C3} * \text{D3}$. The cell D4 will be completed by copying the cell D3. Other approximated values v_i of velocity v will be obtained by copying the line 4. In columns E, F there is expressed an exact solution obtained analytically. Algorithm of the solution of the differential equation $v'(s) = f(s, v)$ within the interval $\langle s_0, s_0 + n.h \rangle$ with the step h and the initial condition $v(s_0) = v_0$ according to the 2nd order Runge-Kutta method can be expressed in a simplified form that can be seen in table 1. If we utilise the relations of the mentioned algorithm and rewrite them into Excel cells, we will obtain a discrete solution to the equation (2), i.e. approximated values of velocity v_i at the distance s_i . In fig. 1 they are pointed up in columns G, H; because of the lucidity, they are not plotted in graph in fig.2. Comparing the approximated values v_i at the distance s_i to the accurate values obtained analytically we can see that Euler's method provides less accurate results.

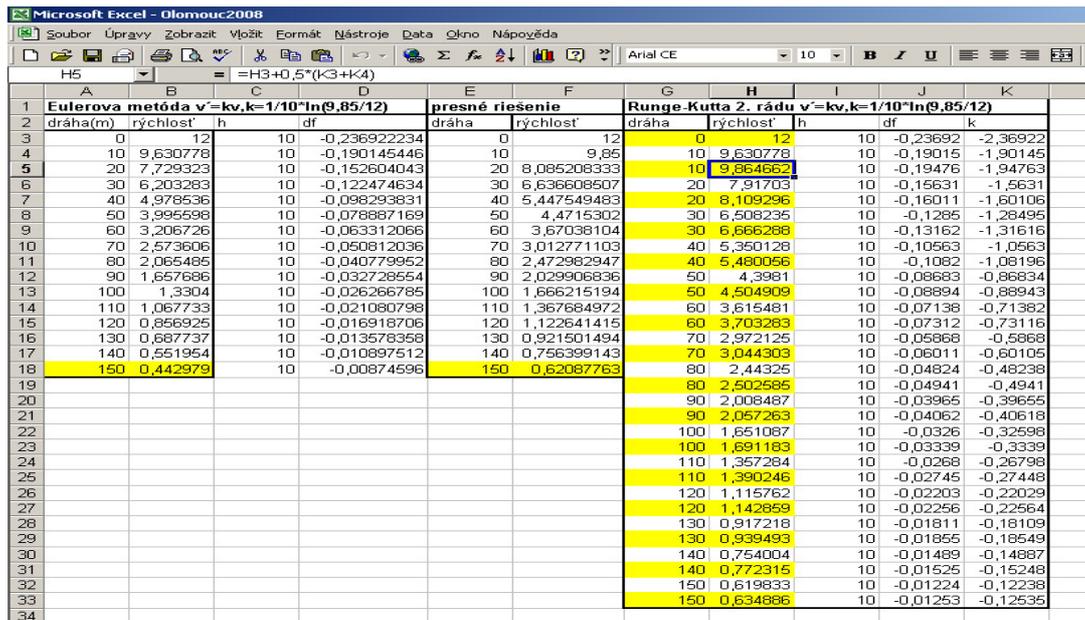


Figure 1. Numerical solution to the differential equation of the wind velocity reduction in Excel

s_i - path	v_i - velocity	k_i
s_0	v_0	$k_0 = h \cdot f(s_0, v_0)$
$s_0 + h$	$v_0 + k_0$	$k_1 = h \cdot f(s_0 + h, v_0 + k_0)$
s_1	$v_1 = v_0 + 0,5 \cdot (k_0 + k_1)$	$k_2 = h \cdot f(s_1, v_1)$
$s_1 + h$	$v_1 + k_2$	$k_3 = h \cdot f(s_1 + h, v_1 + k_2)$
s_2	$v_2 = v_1 + 0,5 \cdot (k_2 + k_3)$	$k_4 = h \cdot f(s_2, v_2)$
$s_2 + h$	$v_2 + k_4$	$k_5 = h \cdot f(s_2 + h, v_2 + k_4)$
...
s_n	$v_n = v_{n-1} + 0,5 \cdot (k_{2n-2} + k_{2n-1})$	

Table 1. 2nd order Runge-Kutta Method

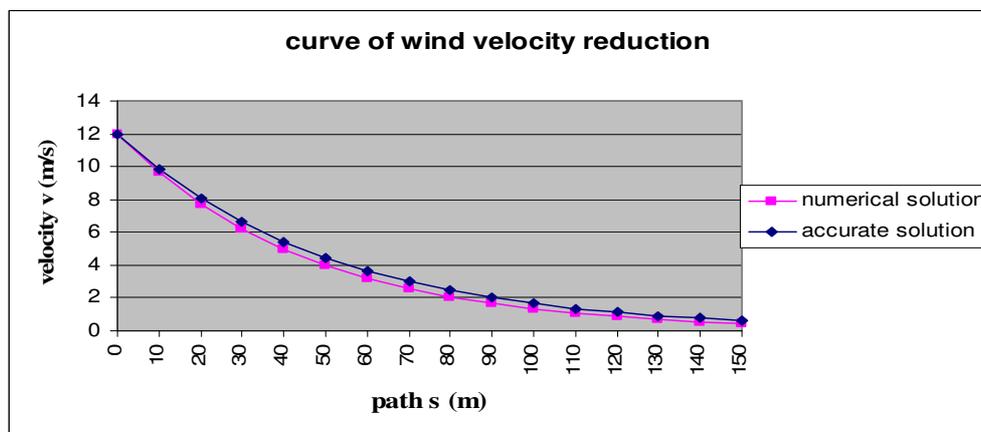


Figure 2. Integral curve of the solution to the differential equation of the wind velocity reduction

4. Conclusion

When verifying the effectiveness of MS Excel utilisation as a software support with the applications of differential equations, for example within numerical mathematics teaching, MS Excel program in general proved as a suitable means. Its advantage consists in its availability as well as in the fact that students are very familiar with MS Excel environment. Most students are able to work with cells (copy, shift...), formulas (to record a function prescription), or to create graphs. Teaching it to the students at practical classes is not time consuming either. Based on experience, the following procedure has been approved - a classical explanation, a given task solution, verification if the students cope with the algorithm, utilization of a computing technique with a suitable mathematical software. ICT implementation to the numerical mathematics teaching enables not only to calculate more tasks, but also compare individual methods from the viewpoint of error estimation, promptness, convergence and suitability of the given method. Another undoubted advantage is a graphical interpretation of numerical solution results with a computing support. However, the goal remains that students cope with and master numerical methods to such an extent that they are able to program them independently in any suitable environment of a mathematical software.

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