# Asymptotic behaviour of nonlinear integral equations

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Consider the nonlinear equation of the form

$$u^p(t) = f(t) + \int_0^t k(t,s)g(s,u(s))ds, \tag{1}$$

where $f : R_+ \to R,\ k : R_+ \times R_+ \to R,\ g : R_+ \times R \to R$ are continuous functions and $p > 1$ is a constant. Okrasinski[1] studied the problem of existence and uniqueness of solutions of equation (1) in the form

$$u^p = k * u + f, \quad p > 1,$$

where $k, f$ are known smooth functions depending on physical parameters. Pachpatte[2] investigated the boundedness and asymptotic behaviour of solutions of (1) by using inequalities derived by himself in [2].

Now suppose $u(t) \geq 0,\ g \geq 0,\ r_i \geq 0,\ i = 1, 2, \ldots, n-1$ are continuous functions defined on $R_+$ and let $p > 1$ be a constant.
Put

$$\begin{aligned} M[t, r, g(t_n)u(t_n)] &= M[t, r_1, \ldots, r_{n-1}, g(t_n), u(t_n)] \\ &= \int_0^t r_1(t_1) \int_0^{t_1} r_2(t_2) \ldots \int_0^{t_{n-2}} r_{n-1}(t_{n-1}) \int_0^{t_{n-1}} g(t_n)u(t_n)dt_n dt_{n-1} \ldots dt_2 dt_1. \end{aligned}$$

In the following it is assumed that every solution $u(t)$ of (1) exists on $R_+$.

**Theorem 1** [3]. *Assume*

$$u^p(t) \leq c + M[t, r, g(t_n), u(t_n)],$$

*for $t \in R_+$, where $c \geq 0$ is a constant.*
*Then*

$$u(t) \leq [c^{(p-1)/p} + ((p-1)/p)\ M[t, r, g(t_n)]]^{(p-1)/p}.$$

Now by Theorem 1 with $n = 1$ we can formulate the following result:

**Theorem 2** *Suppose*

$$|f(t)| \leq c_1 e^{-pt}, \quad |k(t,s)| \leq h(t)e^{-pt}, \quad |g(t,u)| \leq r(t)|u|,$$

*where $c_1$ is a nonnegative constant, $r : R_+ \to R_+,\ h : R_+ \to R_+$ are continuous functions and*

$$\int_0^\infty h(s)r(s)e^{-s}ds < \infty.$$

*Then the solution $u(t)$ of (1) is asymptotic stable.*

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## References

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