Application of the domain decomposition method BDDC (Balancing domain decomposition with constraints) to stress analysis of constructions

Jakub Šístek[1], Marta Čertíková[1], Jaroslav Novotný[1], Pavel Burda[1], and Alexandr Damašek[2]

[1] Department of Mathematics, Czech Technical University in Prague, Karlovo náměstí 13, CZ-121 35 Praha 2 , Czech Republic,
jakub.sistek@fs.cvut.cz, marta.certikova@fs.cvut.cz, novotny@bivoj.it.cas.cz, burda@fsik.cvut.cz

[2] Institute of Thermomechanics, Academy of Sciences of the Czech Republic, Dolejškova 5, CZ-182 00 Praha 8, Czech Republic,
damasek@it.cas.cz

Abstract

We deal with the Balancing Domain Decomposition with Constraints method (BDDC) applied to 3D elasticity. Theoretical framework of the method is recalled in the first part. This is done for problems of linear elasticity covering the case of three dimensions. The second part of the paper discusses practical aspects of the method. Implementation of the method on modern computers is outlined, and possible approaches to parallelization are sketched. Several numerical results of practical problems of structural analysis are presented.

The Balancing Domain Decomposition with Constraints method (BDDC) of C.R. Dohrmann and J. Mandel was originally described in terms of matrices and vectors (see papers [2], [3], [4]).

We describe here the BDDC algorithm for 3D problem of linear elasticity using analytic framework and main ideas from [1] and [5].

The theory was implemented and applied to several problems arising from structural analysis of mechanical components. Results of these computations as well as details on parallel implementation are given in the end of the paper.

BDDC is regarded as a preconditioner for the conjugate gradient method (PCG) iterating on the problem restricted to the interface by static condensation.

Solving the global problem via BDDC method, three main subproblems might be identified. These are the *coarse problem* and *subdomain problems* which form the preconditioner, and *static condensation problems* for each subdomain.

While the PCG method seems to be the right choice for solving interface problems of moderate to large size in applications with symmetric positive definite matrices, one can easily derive several "modifications" of the code combining various established methods of the numerical linear algebra (direct or iterative) for these subproblems. The right choice of the method in this case

might depend on the size and set-up of the problem, i.e. number of subdomains, H/h ratio, structure of interface, etc. An optimal (but complex) implementation might choose adaptively the most suitable method for a given case. However, we restricted ourselves to using the *frontal method* for all three subproblems.

Matrix multiplication within PCG method, where these are needed, is done via backsubstitutions in frontal method and resolving reaction forces in interface nodes of subdomains. In similar fashion, the augmented matrix with constraints is replaced by the prescription of fixed variables (only this simple type of constraints is considered).

Message Passing Interface (MPI) library was chosen as the parallel environment.

However, the size of coarse problem grows and becomes a “bottleneck” of similar methods for very large problems. Therefore, solving the coarse problem in a parallel way is an attractive option for modern domain decomposition implementations.

The algorithm has been ported and optimized on several parallel computers. It has been tested on various problems of linear elasticity in 2D and 3D. Although the program needs further improvement, it has already performed well and reached promising results.

Acknowledgement: This work has been supported by the grant no. 1ET 400760509 of the Information Society Program of the Czech Academy of Sciences and by the State Research Project no. MSM 6840770010.

We acknowledge the help of Svatopluk Pták, who kindly provided his computational mesh. We are also indebted to Jan Mandel and Bedřich Sousedík for fruitful private communication.

References

1. Brenner, S., Sung, L.Y.: BDDC and FETI-DP without matrices or vectors. *http://www.math.sc.edu/~fem/papers/BDDC.ps*, 2005
2. Dohrmann, C.R.: A preconditioner for substructuring based on constrained energy minimization. SIAM Journal on Scientific Computing **25** (2003) 246–258
3. Mandel, J., Dohrmann, C.R.: Convergence of a balancing domain decomposition by constraints and energy minimization. Numer. Linear Algebra Appl. **10** (2003) 639–659
4. Mandel, J., Dohrmann, C.R., Tezaur, R.: An algebraic theory for primal and dual substructuring methods by constraints. Applied Numerical Mathematics **54** (2005) 167–193
5. Le Tallec, P.: Domain decomposition methods in computational mechanics. Computational Mechanics Advances **1** (1994) 121–220
6. Toselli, A., Widlund, O.: Domain Decomposition Methods - Algorithms and Theory. Springer-Verlag, Berlin Heidelberg, 2005