On Mathematical Modeling of Composites

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Abstract

Mathematical and physical modeling of a behavior of two-phased composite materials lead to a problem that is not nowadays correctly and theoretically explained and cleared up. Algorithms trying to find a right solution for its properties break down, because of its uncertainty from a viewpoint of probability and real distributions. The aim of this article will be to introduce one of many statistical descriptors used to compare various random samples generated by different algorithms, introduced e.g. in [1].

1 Introduction

Composite materials consist of at least two different phases, particularly two-phase fiber composite consists of the so called stiffening phase in the shape of a fiber included into the second one called matrix. By various distribution of fibres we can develop materials with special properties, that is the reason, why they are intensively used in engineering.

Mathematical method, called *homogenization* supposes periodic distribution of the fibres, which is not true in reality. A favorite approach to describe real composites is *spatial statistic* and its tools, see e.g. [2]. We will introduce so called *Ripley's K-function* that belongs to the so called second-order methods for description of spatial data.

2 Ripley's $K(t)$-function

Ripleys $K(t)$ function is a tool for analyzing a completely mapped spatial point processes data, i.e. data on the locations of events. Here we describe $K(t)$ for two dimensional spatial data. Completely mapped data include the locations of all events in a predefined study area. The $K(t)$ function is defined as

$$K(t) = \lambda^{-1}\mathbf{E}[\text{ number of events within distance } t \text{ of a randomly chosen event }],$$

where λ is the density (number per unit area) of events. $K(t)$ does not uniquely define the point process in the sense that the two different processes can have the same $K(t)$ function. Also, processes with the same $K(t)$ function may have different nearest-neighbor distribution function. Nevertheless, the K function is the basis of routine tools (for descriptive and testing purposes) widely used in the analysis of spatial processes. For many point processes the expectation in the numerator of the $K(t)$ function can be analytically evaluated, so the $K(t)$ function can be written in a close form. The simplest and most commonly used, is $K(t)$ for a homogeneous Poisson process (complete spatial randomness):

$$K(t) = \pi t^2.$$

Values of $K(t)$ for a process are often compared with those for the Poisson process. Values larger or smaller than πt^2 respectively indicate a more clustered or more regular process than the Poisson process. The most commonly used estimator for this function is

$$\hat{K}(t) = \frac{|A|}{n^2} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w(l_i, l_j)^{-1} I(d_{ij} < t),$$

where $|A|$ is the size of our area(sample), n is the count of fibres in A, d_{ij} is the distance between the $i-$th and $j-$th points, and $I(x)$ is the indicator function. The weight function $w(l_i, l_j)$ provides the edge correction. It has the value of 1 when the circle centered at l_i and passing through the point l_j (i.e. with a radius of d_{ij}) is completely in the study area (i.e. if d_{ij} is larger than the distance from l_i to at least one boundary). If part of the circle falls outside the study area, then $w(l_i, l_j)$ is the proportion of the circumference of that circle that falls in the study area. The effects of edge corrections are more important for large t, because large circles are more likely to be outside the study area. In the next figure is shown an example of

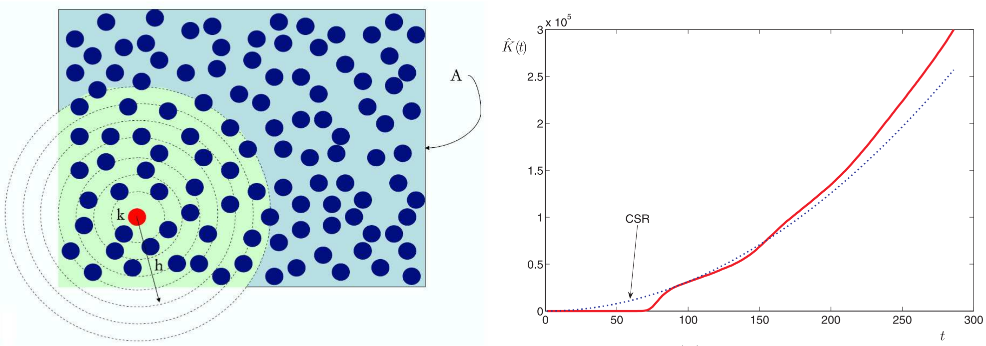

Figure 1: *A figure related to explanation to the Ripley's $K(t)$ function and its estimation.*

the Ripleys K-function for real composite. The shape of the function was computed as a mean of 15 real samples. The other line represent complete spatial random sample (CSR).

3 Conclusions

In this article we showed a difference between real samples of composite and idealized, complete spatial random sample. To the testing we used Ripley's $K-$ function, which is one of the most used function for testing in spatial statistic problems.

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