Some remarks on non-integer differential and integral calculus

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Abstract

Differential and integral calculus belongs to basic courses of mathematics and everybody understands geometrical and physical meaning of derivative or integral. But, hardly anybody can imagine for example derivative of order 1/2 or even of non-rational order. The branch of mathematics which generalizes calculus to non-integer case is known under the term "fractional calculs" although this name is little bit misleading.

The beginings of the fractional calculus fall to the end of the 17^{th} century, when Leibniz made a note in a correspondence with L'Hospital about meaning of the derivative of order one half. Then, during three centuries, the theory of non-integer derivatives had been developed mainly from a theoretical point of view. Applications appeared just before a few decades, e.g. in material engineering, control of dynamical systems, chemistry and physics. Special functions – mainly Euler's Gamma function – play essential role in fractional calculus. We also refer to Beta, Mittag-Leffler, Wright functions that are useful – a brief overview can be found in [1],[5].

Several approaches how to define fractional (or non-integer) derivatives (integrals) , e.g. Grünwald-Letnikov, Cauchy, Caputo, Riemann-Liouville and others (see e.g. [2], [3],[4],[5]), are known. Last two mentioned are most often used in applications because of relatively weak assumptions on differentiated functions.

The contribution focuses on approach by Grünwald-Letnikov and Riemann-Liouville as well as on motivation for these definitions. Several examples and comments are added. The text is intended to be a brief overview for readers that are not familiar with the topic, so instead of proofs, rather some explanatory remarks are presented.

The full text is presented in the electronic part of proceedings.

Acknowledgement. Published results were acquired using the subsidization of the Ministry of Education, Youth and Sports of the Czech Republic, research plan MSM 0021630518 "Simulation modelling of mechatronic systems".

Reference

[1] Andrews, G. E., Askey, R., Roy, R.: *Special functions*, Enc. of Math. and its Appl., Cambridge University Press, Cambridge, (1999).

[2] Kilbas, A. A., Srivastava, H. M., Trujillo, J. J., *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, (2006).

[3] Li, C., Deng, W., *Remarks on fractional derivatives*, Appl. Math and Comp. 187, 777–784, (2007).

[4] Ortigueira, M. D., *A coherent approach to non-integer order derivatives*, Signal Processing 86, 2505–2515, (2006).

[5] Podlubny, I.: *Fractional Differential Equations*, Academic Press, New York, (1999).