Some remarks to n-dimensional s-map

Oľga Nánásiová, Katarína Trokanová, Ivan Žembery
Slovak University of Technology in Bratislava, Faculty of Civil Engineering,
Dept of Math., Radlinského 11, 813 68 Bratislava
e-mail: olga.nanasiova@stuba.sk, trokan@math.sk, ivan.zembery@stuba.sk

In [1] has been shown, that it is possible to constract such sequence of non symmetric s-maps [2], which converges to symmetric s-map. The proof was based on next proposition which is independent on s-maps.

Proposition. *Let X be a non empty set and let $f : X \times X \to R$ be a function . Let the following sequence be defined $\{f_n\} n = 0, 1, 2, ...$ by the way $f_0(x, y) = f(x, y)$ and $f_{n+1}(x, y) = kf_n(x, y) + (1 - k)f_n(y, x)$. where $k \in (0, 1)$. Then*

$$\lim_{n\to\infty} f_n(x, y) = \frac{1}{2}(f(x, y) + f(y, x)).$$

It can be write in matrix form in the following way:

$$\begin{pmatrix} f_n(x,y) \\ f_n(y,x) \end{pmatrix} = \begin{pmatrix} k & 1-k \\ 1-k & k \end{pmatrix}^n \begin{pmatrix} f(x,y) \\ f(y,x) \end{pmatrix}.$$

We have proved $\lim_{n\to\infty} \begin{pmatrix} k & 1-k \\ 1-k & k \end{pmatrix}^n = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

This way of formulation of the problem gives possibility to generalization for k-dimensional s-maps.

Hypothesis:

$$\lim_{n\to\infty} \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & a_{k-1} & a_k \\ a_2 & a_3 & a_4 & \cdots & a_k & a_1 \\ a_3 & a_4 & a_5 & \cdots & a_1 & a_2 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ a_{k-1} & a_k & a_1 & \cdots & a_{k-3} & a_{k-2} \\ a_k & a_1 & a_2 & \cdots & a_{k-2} & a_{k-1} \end{pmatrix}^n = \begin{pmatrix} \frac{1}{k} & \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} & \frac{1}{k} \\ \frac{1}{k} & \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} & \frac{1}{k} \\ \frac{1}{k} & \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} & \frac{1}{k} \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ \frac{1}{k} & \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} & \frac{1}{k} \\ \frac{1}{k} & \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} & \frac{1}{k} \end{pmatrix}$$

for $a_i \in< 0, 1)$ and $\sum_{i=1}^{k} a_i = 1$.

Acknowledgement. This work was supported by Science and Technology Assistance Agency under the contract No. APVT-20-003204, VEGA-1/3014/06, VEGA-1/3321/06, VEGA-1/3006/06

References

[1] Nánásiová, O., Trokanová K., Žembery I..(2007) *Commutative and non commutative s-maps*, Forum Statisticum Slovacum, **2/2007**, 172 – 177.

[2] Nánásiová, O.: Map for Simultaneous Measurements for a Quantum Logic. Int. Journ. of Theor. Phys., *42* (2003), pp. 1889-1903.