Stability of linear convolution systems

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We consider the linear integro-differential system of convolution type

$$x'(t) = Ax(t) + \int_0^t K(t-s)x(s)ds, \tag{1}$$

where A is $n \times n$ constant matrix and $K\colon R^+ \to R^{n\times n}$ is a continuous matrix.

Some results concerning asymptotic behavior of linear and nonlinear systems of Volterra integro-differential equations are established in [1, 2].

Let $x(t) = x(t, t_0, \phi)$ be any solution of (1) with the initial function $\phi(t)$ on $0 \le t \le t_0$ for some $t_0 > 0$. Then we have

$$x'(t) = Ax(t) + \int_{t_0}^t K(t-s)x(s)ds + \int_0^{t_0} K(t-s)\phi(s)ds.$$

Now using the variation of parameters formula we obtain

$$x(t) = Z(t-t_0)\phi(t_0) + \int_{t_0}^t Z(t-s)\left[\int_0^{t_0} K(s-\tau)\phi(\tau)d\tau\right]ds,$$

where Z is the resolvent of the kernel K.

Let the function $p(t)$ defined by

$$p(t) := \int_0^\infty \left|\int_0^t Z(t-\tau)K(\tau+\sigma)d\tau\right| d\sigma$$

exists and be finite for all $t \ge 0$.

Before proceeding further we need a suitable definition of stability.

Definition 1 *The trivial solution of* (1) *is stable if for any $\epsilon > 0$ and $t_0 \in R^+$ there exists $\delta = \delta(t_0, \epsilon) > 0$ such that*

$$|\phi|_{t_0} = \max_{0\le s\le t_0} |\phi(s)| < \delta$$

implies $|x(t, t_0, \phi)| < \epsilon,\ t \ge t_0$.

Based on this definition other stability notions can be defined.

Theorem 1 *Suppose that $K \in L^1(R^+)$. Then the following statements are equivalent:*

(i) the equilibrium solution $x \equiv 0$ of (1) is uniformly asymptotically stable;

(ii) the equilibrium solution $x \equiv 0$ of (1) is exponentially stable;

(iii) there exist positive real numbers M and μ such that $|Z(t)| \leq Me^{-\mu t}$ for all $t \geq 0$;

(iv) $Z(t) \in L^1(R^+)$;

(v) the solutions of (1) are uniformly bounded;

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References

[1] Z. Šmarda. *Asymptotic stability of solutions of nonlinear Volterra integrodifferential equations,* Department of Mathematics Series, 14(1)(2007), 245-249, ISSN 1214-4681.

[2] Z. Šmarda. *On stability of linear integrodifferential equations,* Proceedings of IV. International Conference on Soft Computing Applied in Computer and Economic Enviroment, Kunovice, European Polytechnical Institute, (2006), 163-168, ISBN80-7314-5.