Actions of Join Spaces of Continuous Functions on Hypergroups of Second-Order Linear Differential Operators

Jan Chvalina
Brno University of Technology, Faculty of Electrical Engineering and Communication
Department of Mathematics
e-mail: chvalina@feec.vutbr.cz

Jiří Moučka
Brno University of Defense, Faculty of Economics and Management
Department of Econometrics
e-mail: jiri.moucka@unob.cz

Dedicated to Professor František Neuman on the occasion of his 70th birthday

Abstract

One from classical constructions of algebraic binary hyperstructures (semihypergroups and hypergroups) from ordered algebraic systems is based on a certain Lemma on principal ends generated by products of pairs of elements – shortly termed as Ends-Lemma. First version of this auxiliary lemma was obtained in the monography [3]. The mentioned lemma is applied not only in case of hyperstructures created from ordered or quasi-ordered semigroups and groups but also in constructions based centralizers of various transformations (in partricular endomorphism monoids of mono-unary algebras of various functions or operators). Moreover constructions of this type yield the way to obtain semihypergroups and hypergroups of linear differential operators – ordinary differential operators of a given order n or partial differential operators and mentioned constructions allow also to create hyperstructures of integral operators of Fredholm or Volterra-type – cf. [2], [5].

In the present contribution we construct actions of commutative transposition hypergroups i.e. join spaces created from rings of continuous and smooth functions of a given class on semihypergroups or hypergroups of second order linear ordinary differential operators. These constructed structures are in fact discrete dynamical systems with a phase (additive) hypergroups of continuous and smooth functions and phase set formed by the above mentioned differential operators. As a suitable synonyma we can use a multiautomaton without output function, where the transition function or next state funcion satisfies so called Generalized Mixed Asociativity Condition (GMAC) – see the complete contribution. It is to be noted that the systematic investigation of transformations of ordinary differential equations using algebraic tools and methods is going back into fifties which has been initiated by Professor Otakar Borůvka and his school, [7]. In this connection recall from the paper [6] of one of his outstanding successors – Professor František Neuman, to whom we dedicate our contribution: "Algebraic, topological and geometrical tools together with the methods of the theory of dynamical systems and functional equations make it possible to deal with problems concerning global properties of solutions by contrast to the previous local investigations and isolated results". The main part (4) of the contribution contains the isomorphism theorem between actions (discrete dynamical systems – called in the last Borůvka's paper [1] as algebraic spaces with operators) of an additive join space of real functions of the given class C^k on the phase space formed by second-order linear ordinary differential operators and actions of the mentioned functional join space of hypergroups of transformation operators which are restrictions on intervals of the real line of Laplace-transform images of the Volterra-type integral operators with difference kernels. Notice that the just mentioned classical Laplace transform, which is a part of curricula of mathematical subjects – in particular at faculties of

electrical engineering of universities of technology, enables the construction of an embedding of the above mentioned Volterra integral operators in the hypergroup of certain complex transformations generalizing affine transformations of vector function spaces. These facts serve as motivating factors of presented constructions.

References

[1] O. Borůvka. *Algebraic spaces and their realization by differential equations I, II.* Text of "Seminář o diferenciálních rovnicích", PřF UJEP, 1988, 35pp (in Czech).

[2] Š. Hošková, J. Chvalina, P. Račková. *Transposition hypergroups of Fredholm integral operators and related hyperstructures.* AHA 2005, Babolsar, Iran, 2005, 23pp (in print).

[3] J. Chvalina. *Functional Graphs, Quasi Ordered Sets and Commutative Hypergroups.* Brno, Masaryk University, 1995 (in Czech).

[4] J. Chvalina, L. Chvalinová. *Join spaces of linear ordinary differential operators of the second order.* Colloqium on Differential and Difference Equations, CDDE 2002. In: Folia Fac. Sci. Nat. Univ. Masarykianae Brunesis, Mathematica 13, MU Brno, 2003, 77–86.

[5] J. Chvalina, M. Novák. *Laplace-type transfromation of a centralizer semihypergroup of Volterra integral operators with translation kernel.* XXIV Internat. Colloqium on the Acquisition Process Management, Proc. of Contributions, Brno, University of Defense, 2006, 9pp.

[6] F. Neuman. *From local to global investigations of linear differential equations of the n-th order.* Jahrbuch Überblicke Mathematik 1984, 55–80.

[7] F. Neuman. *Global Properties of Linear Ordinary Differential Equations.* Mathematics and its Applications, East European Series 52, Kluwer Academic Publishers (with Academia Praha) Dordrecht–Boston–London 1991.