On Modeling Uncertainty in Input and Output Data

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1 Introduction

Almost all (if not all) models are burdened with uncertainty. Even if we set aside the difficulties in choosing the adequate level and complexity of a mathematical model, the problem of uncertainty in the numerical values of input parameters is omnipresent.

In civil or mechanical engineering, for instance, the knowledge of material parameters is essential. Their numerical values can be found in handbooks but, usually, only as crisp numbers without details about their origin. Some basic statistical features (as histograms or standard deviations) of the fundamental material parameters can be found in extensive or specialized handbooks, but these are hardly available to ordinary analysts.

2 Modeling uncertainty

2.1 Stochastic methods

If probabilistic characteristics of model inputs are known, then various stochastic approaches can be applied. Among them, the Monte Carlo method is widely used. It is based on generating a number of samples of input values, calculating the model response, and analyzing it statistically to infer fundamental probabilistic characteristics of the model outputs.

Stochastic finite elements are also an option. In this approach, the random quantity is approximated by its truncated expansion, see [3, 4].

It is also possible to transform stochastic equations to deterministic equations in higher dimensions. The extra dimensions are related to the truncated expansion of the random variable, see [1].

The stochastic methods can deliver a highly informative description of the model output behavior. However, they assume rather "strong" information about the model inputs behavior. Such information is not always available or is too expensive.

2.2 Worst scenario method

In the stochastic approach, the inputs are weighted by their probability. In the worst scenario method, the inputs are not weighted at all. Under the assumption of (a) the compactness of the set of admissible (uncertain) parameters and (b) the continuous dependence of the quantity of interest (model output) on input parameters, the existence of the extremes (the worst and the best scenarios) among admissible output values can be proved, see [5], where approximate scenarios and their convergence are also studied. In practice, however, the equality of inputs is not common and data weighting is often possible. This is why the coupling of the worst scenario method with other methods is worth considering.

2.3 Dempster-Shafer approach

This method weights (sub)sets of admissible parameters, see [2, 6]. The weight expresses the evidence that we have for particular (sub)sets of input data. To extend the weight into model outputs, the worst scenario method is applied to each weighted (sub)set of inputs.

2.4 Fuzzy set approach

In this approach, the values of the admissible parameters are weighted by a membership function, see [7, 8], and the goal of the uncertainty analysis is to infer the membership function of the quantity of interest. In this process, again, relevant worst scenario problems has to be solved.

3 Conclusions

Since the worst scenario method appears as a part of other methods for modeling uncertainty and its propagation through models, its algorithmization deserves much attention. Fortunately, all necessary ingredients are at hand. Indeed, the core of the method is nothing else than constrained global optimization. Consequently, relevant tools such as sensitivity analysis, genetic algorithms, and methods based on the gradient or the optimality conditions can be used in the search for the best and the worst scenarios.

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