Sufficient conditions for stability of discrete equations with delay

Oleg Anashkin
Taurida National University, Simferopol', Ukraine,
e-mail: anashkin@crimea.edu

Let $J[a,b] \subset \mathbb{Z}$ be a set of integer numbers on the segment $[a,b] \subset \mathbb{R}$. For given integer p we consider a linear space $\mathfrak{M}_p = \{\varphi\colon J[-p,0] \to \mathbb{R}^n\}$ equipped with the norm $\|\varphi\| = \max\{|\varphi(s)| : s \in J[-p,0]\}$, where $|\cdot|$ is a norm in $\mathbb{R}^n$. Obviously, each element φ of space $\mathfrak{M}_p$ corresponds to a $(n \times (p+1))$-matrix $\varphi = (\varphi(-p), \dots, \varphi(0))$. For given sequence $x : \mathbb{Z} \to \mathbb{R}^n$, $k \mapsto x(k)$, denote by $x[k]$ an element of $\mathfrak{M}_p$ defined as $x[k](s) = x(k+s)$, $s = -p, \dots, 0$.

Our goal is to present several theorems on sufficient conditions for stability of the zero solution of nonlinear delay difference equation of the form

$$\Delta x(k) = f(k, x[k]), \quad k = \sigma, \sigma+1, \dots, \tag{0.1}$$

where $x = (x_1, \dots, x_n)$, $\Delta x(k) = x(k+1) - x(k)$. For each given $k \in \mathbb{Z}$ function $f : \mathbb{Z} \times \mathfrak{M}_p \to \mathbb{R}^n$ is defined in the region $\mathfrak{B}_H^p = \{\|\varphi\| < H\} \subset \mathfrak{M}_p$ and there are constants $M > 0$ and $d_0 > 1$ such that

$$|f(k,\varphi)| \le M\|\varphi\|^{d_0}, \quad k \in \mathbb{Z}, \quad \varphi \in \mathfrak{B}_H^p.$$

Let us consider a set $\mathfrak{A}_R = \{\varphi \in \mathfrak{M}_p : \|\varphi\| \le R|\varphi(0)|\}$ with a parameter $R \ge 1$. Using some properties of solutions to (0.1) with respect to set $\mathfrak{A}_R$ we are able to enlarge a set of appropriate Lyapunov fuctions for a concrete equation of the form (0.1). In particular, the presented approach gives an opportunity to formulate sufficient conditions on asymptotic stability in terms of non-decrescent Lyapunov functions. Some illustrative examples are given.

References

[1] O. V. Anashkin, *Sufficient conditions on stability for a class of nonlinear delay difference equations,* Uchenye zapiski TNU, **19(58)** (2006), 12–19.