Numerical reconstruction of a circle from its photographical image at subpixel precision

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The analysis of gray-scaled photographical images for circular domains is a serious problem in image processing, namely in applications in astronomy, physics, biology, quality control and metrology, etc. One needed application at the Faculty of Civil Engineering of the Brno University of Technology is the (rather cheap) monitoring of displacement of parts of building constructions in time, related to some reference configuration, using the standard camera, at subpixel precision. The full text (26 pages) of this paper, including extensive references, can be found in the CD-ROM Proceedings; here only its basic ideas are sketched.

The methods, discussed in the literature, can be divided into two big groups: optimization and voting/clustering. The first group contains namely least squares fitting, moment of inertia optimization and genetic algorithms. The second group incorporates Huge transform, random sample consensus, algorithms based of fuzzy logic and algorithms based on competitive learning. However, other criteria are possible, too: some methods prefer robustness, other methods computational efficiency, including either low CPU time (important namely in real-time applications) or small memory requirements or make use of possibilities of parallel computing, etc.; the result is every time some compromise.

An important part of the correct ellipse detection, needed in both groups of methods, is the good choice of approximating points from a gray-scaled two-dimensional map, consisting of a finite number of rectangles with constant intensities between black and white. Quick (especially real-time) applications make use of some intuitive searching for maximal derivatives, preceded by certain discretized version of Gaussian smoothing. Better approximations may be exploited using the methods of numerical solution of partial differential equations of evolution, namely a nonlinear diffusion equation of Perona-Malik type. From our field of interest namely the development of geodesic active contours, should be applicable.

The complete study of the problem consists of the following steps: i) the geometrical analysis of the central projection of a circle to a plane for some a priori presribed parameters, making use of the least squares technique, ii) the choice of data smoothing and contour detection approach, iii) the facultative inclusion of the eliptical shape information into contour detection. iv) the analysis of real parameters of calculations and their accuracy and reliability during all measurements.

As an example (in the full text the simplest one) of the software algorithm, based on the proper mathematical analysis, we can demonstrate:

I. initialize the "level set function" ϕ_0, set $k \leftarrow 0$,

II. compute $c_1(\phi_k)$ and $c_2(\phi_k)$ from certain "regularized version" of

$$c_1 = \left(\int_\Omega H(\phi)\,\mathrm{d}\mu\right)^{-1} \int_\Omega gH(\phi)\,\mathrm{d}\mu\,, \qquad c_2 = \left(\int_\Omega (1-H(\phi))\,\mathrm{d}\mu\right)^{-1} \int_\Omega g(1-H(\phi))\,\mathrm{d}\mu$$

where the (discontinuous) Heaviside functions H is smoothened, using the parameter ε, to H_ε (and similarly the Dirac measure δ to δ_ε), and ϕ_k replace ϕ (which generates c_{1k} and c_{2k} instead of c_1 and c_2),

III. to receive ϕ_{k+1}, solve

$$\phi_{k+1} = \phi_k + \tau\delta_\varepsilon(\phi_k)\left(\lambda\,\mathrm{div}\left(\frac{\nabla\phi_{k+1}}{|\nabla\phi_k|}\right) - (g-c_{1k})^2 + (g-c_{2k})^2\right)$$

IV. check whether ϕ_{k+1} is sufficiently close to ϕ_k; if not, set $k \leftarrow k+1$ and go back to II., else accept ϕ_k as final ϕ,

V. initialize old $a, b, \widetilde{x}_0, \widetilde{y}_0, \rho$,

VI. prepare $(x_1, y_1), \ldots, (x_n, y_n)$ by the Newton iterations

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} \leftarrow \begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} p_x(x_i,y_i) & p_y(x_i,y_i) \\ q_x(x_i,y_i) & q_y(x_i,y_i) \end{bmatrix}^{-1} \begin{bmatrix} p(x_i,y_i) \\ q(x_i,y_i) \end{bmatrix}$$

with $i \in \{1, \ldots, n\}$ (p and q are rather complicated, but correctly determined real functions, their indices refer to partial derivatives),

VII. compute new $a, b, \widetilde{x}_0, \widetilde{y}_0, \rho$, using the least squares method and Newton iterations (too formally complicated for the detailed presentation here),

VIII. check whether new and old $a, b, \widetilde{x}_0, \widetilde{y}_0, \rho$ are nearly the same; if not, take new values as old ones and go back to VI.

Here a, b are the real semiaxes of the ellipse, $\widetilde{x}_0$ and $\widetilde{y}_0$ are the transformed coordinates of the centre of the original circle and the remaining parameter ρ is determined by the position of the camera; the symbol g is used instead of "gray-scaled data".

This research was partially supported by the Grant Agency of the Czech Republic, Reg. No. 103/05/0292.