Mercator´s asset in mathematical cartography

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1 Preface

The topic of the talk is the science of map making. It is common knowledge that one cannot press the peel of an orange against a table without tearing them apart. Surprisingly, it was not until the middle of the eighteenth century that this fact was proved mathematically, and by none other than Euler. Had the earth been a cylinder, or a cone, the mapmakers task would have been easier - these surfaces can be flattened without shrinking. But the underlying geometry of a sphere is fundamentally different from that of a plane, consequently, one cannot create a map of the earth that faithfully reproduces all its features.

2 Map projections

To cope with this problem, cartographers have devised a variety of map projections - functions that assign to every point on the sphere an "image" point on the map. The choice of a particular projection depends on the intended purpose of the map - one map may show the correct distance between two points on the globe, another the relative area of countries and yet another the direction between two points.

The simplest of all projections is the cylindrical projection. The most striking feature of the cylindrical projection is the excessive north - south stretching at high latitudes, resulting in a drastic distortion of the shape of continents.

A second projection is the stereographic projection. One can show that the stereographic projection is direction - preserving, or conformal.

For many years it had been believed that a path of constant bearing - known as a loxodrome - is an arc of a great circle. But the Portuguese Pedro Nunes in the sixteenth century showed that loxodrome is actually a spiral curve that gets ever closer to either pole, winding around it indefinitely but never reaching it.

The challenge that faced cartographers in the sixteenth century was to design a map that would show all loxodromes as straight lines. It befell a Flemish mapmaker to come to the mariners rescue.

3 Gerardus Mercator

Gerardus Mercator, by general consensus the famous mapmaker in history, was born in Flanders (now Belgium but then part of Holland) on March 5, 1512. He entered the University of Louvain in 1530 and soon after graduating established himself as one of Europes leading mapmakers.

It was in 1568 that Mercator set himself the task of inventing a new map projection. From the outset he was guided by two principles: the map was to be laid out on a rectangular grid, with all circles of latitude represented by horizontal lines parallel to the equator and equal to it length, and all meridians showing as vertical lines perpendicular to the equator. And the map would be conformal, for only such a map could preserve the true direction between any two points on the globe.

Now on the globe, the circles of the latitude decrease in size as their latitude increases. But on Mercators map these same circles show as horizontal lines of equal length. Consequently, each parallel on the map is stretched horizontally by a factor that depends on the latitude of that parallel.

And now Mercator was ready to produce his trump card: in order for the map to be conformal, the east - west stretching of the parallels must be accompanied by an equal north - south stretching of the spacing between the parallels, and this north - south stretching progressively increases as one goes to higher latitudes. This is the key principle behind his map.

Mercator died in Duisburg on December 2, 1594. Yet his most famous achievement, the map that bears his name, was not immediately embraced by the maritime community. The fact that Mercator had not given a full account of how he had progressively increased the distance between the parallels only added to the confusion.

Figure 1: Gerardus Mercator.

4 Edward Wright

It was left to Edward Wright (1560 - 1615), an English mathematician, to give the first accurate account of the principles underlying Mercators map. Wright used numerical integration to evaluate $\int_0^\phi \sec\phi$. Let us follow his plan.

5 Epilogue

We are now in a position to write down the coordinates (x, y) of a point P on Mercators map in terms of longitude λ and latitude ϕ of corresponding point P on the globe. The difference equation $\Delta x = R\Delta\lambda$ has the obvious solution $x = R\lambda$. We thus have

$$x = R\lambda,\ y = R\int_0^\phi \sec t\,.$$

References

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