Singular periodic solutions of perturbed integrodifferential equations

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Consider the following integrodifferential equation

$$\epsilon y'(t) = b(t)y(t) + \int_0^\omega K(t,s)y(s)ds + \epsilon \sum_{n=0}^{m} A_n(t)y^n(t), \tag{1}$$

where $\epsilon \in R$ is a small parameter, $b(t), A_n(t) \in C(R), n = 0, \ldots, m$ are periodic functions with the period ω, $K(t,s) \in C(R \times R)$ is a periodic function with respect to variables t, s with the period ω and $m \in N,\ m \geq 2$.

In many papers [1-7] there have been established sufficient conditions of existence and uniqueness of periodic solutions of ordinary differential and integrodifferential equations under various perturbed terms.

In this paper we will investigate the equation (1) by means of a limit problem and, moreover, we determine a form of the singular periodic solution of (1) depending on the solution of the limit problem.

Recall that the singular periodic solution of (1) will be called the periodic solution $y(t,\epsilon)$ of (1) with

$$\lim_{\epsilon \to 0} |y(t,\epsilon)| = \infty.$$

Now we put

$$\epsilon = \lambda^{m-1}, \quad x(t) = \lambda y(t).$$

Then the equation (1) has the form

$$\lambda^{m-1}x'(t) = b(t)x(t) + \int_0^\omega K(t,s)x(s)ds + \sum_{n=0}^{m} \lambda^{m-n} A_n(t)x^n(t). \tag{2}$$

From here we obtain the limit problem of (2)

$$b(t)x(t) + \int_0^\omega K(t,s)x(s)ds + A_m(t)x^m(t) = 0. \tag{3}$$

Let $x_0(t)$ be a continuous periodic solution of (3). Denote

$$a(t) = b(t) + mA_m x_0(t)^{m-1}, \quad K_1(t,s) = K(t,s)a^{-1}(t),$$

$$z(t) = -A_{m-1}x_0(t)^{m-1}a^{-1}(t) - \int_0^\omega R(t,s)A_{m-1}(s)x_0^{m-1}(s)a^{-1}(s)ds,$$

where $R(t,s)$ is the resolvent of the kernel $K(t,s)$.

Theorem. *Assume the following conditions:*

(i) $a(t) \neq 0,\ a(t), b(t), A_n(t) \in C^1(R), n = 0, 1, \ldots, m,\ K(t,s) \in C^1(R \times R)$

(ii) the equation (3) *has the periodic solution* $x_0(t)$.

(iii) the kernel $K_1(t,s)$ *is not situated on a spectrum.*

(iv) there exists a periodic solution $\alpha(t,\epsilon)$ *of* (2), $|\alpha(t,\epsilon)| \leq L$ *as* $\epsilon \to 0,\ L \in R,\ L > 0$.

Then there exists the unique singular periodic solution $y(t,\epsilon)$ *of* (1) *such that*

$$y(t,\epsilon) = \epsilon^{\frac{1}{1-m}} \left(x_0(t) + z(t)\epsilon^{\frac{1}{1-m}} + \epsilon^{\frac{2}{1-m}} \alpha(t,\epsilon) \right).$$

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