An Influence of Parameters on the Shape of NURBS Surfaces

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1 Introduction

NURBS (Non-Uniform Rational B-Spline) surfaces became foundation-stone in many CAD systems. Its biggest advantage is unlimited design possibilities with local control. In this paper, it will be shown the geometric essence of the knot vector changes and the influence of the different types of knot vector on the shape of the result surface. We also show the influence of the weight and the position of the control point.

NURBS surfaces are implemented in many graphic software (e.g. Maya, FormZ, Blender). Users can modify free-form shaped surfaces with a position of the control points, its tangents and also with the knot vectors. In this article we report on the geometric essence of it. Base functions will be drawn with Maple and result surfaces will be drawn with FemDev – the testing environment for RFEM 3D.

2 B-Spline, NURBS Surface

Definition 1. Knot vector t is a non-decreasing sequence of the positive real numbers $\mathbf{t} = (t_0, t_1, \dots t_n)$. The knot vector is called **uniform**, when $t_{i+1} - t_i = t_i - t_{i-1}$ for $i = 1, 2, \dots, n-1$, else the knot vector is called **non-uniform.**

Definition 2. Let $\mathbf{t} = (t_0, t_1, \dots t_n)$ be a knot vector. **B-spline function** N_i^k of degree k is defined by

$$N_i^0(t) = \begin{cases} 1 & \text{for} \quad t \in \langle t_i, t_i + 1) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$N_i^k(t) = \frac{t - t_i}{t_{i+k} - t_i} N_i^{k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} N_{i+1}^{k-1}(t), \tag{2}$$

where $0 \le i \le n-k-1, 1 \le k \le n-1, \frac{0}{0} := 0$. B-Spline function is often called **base function**

for NURBS curves and surfaces.

3 The influence of the position and the weight of the control points

The most useful is changing of the control points. User can modify the final surface with drag and drop. The adventage is that this modification is local. The point P_{ij} changes the surface for the intervals $\langle u_i, u_{i+n+1}) \times \langle v_i, v_{i+m+1})$, where m, n are the surface degrees.

Every control point on NURBS surface has its weight. This number shows, how powerful the point will be. Implicitly, all weight are equal to one. For weights smaller than one has control point less power. For weight bigger than one, the power grows up.

4 The influence of the knot vectors

Base functions for NURBS surfaces are products $N_i^n N_j^m$ for $i = 0, \ldots, q$ and $j = 0, \ldots, r$.

$$\sum_{i=0}^{q} \sum_{j=0}^{r} N_i^n N_j^m = N_0^n N_0^m + N_0^n N_1^m + \ldots + N_0^n N_r^m + N_1^n N_0^m + \ldots + N_q^n N_r^m \tag{3}$$

Every point P_{ij} has the different base function $N_i^n(u) N_j^m(v)$ for parameters (u, v). The value of the base function indicates percentage influence of this point to the surface. This number is generally in the interval $\langle 0, 1\rangle$ because of B-Spline properties - nonnegativity and partition of unity (see (3)).

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