A TOPOLOGIZATION OF POINTLESS CAUSALITY STRUCTURES

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Abstract. During the presentation we will discuss a reconstruction of a topological structure of a generalized spacetime from a corresponding Crane-Christensen causal site via framework theory.

A *causal site*, introduced by Crane and Christensen in [4], is a pointless mathematical structure, given by a set $\mathcal{S}$ of *regions* with two binary relations $\sqsubseteq$, $\prec$, where $(\mathcal{S}, \sqsubseteq)$ is a partial order having the binary suprema $\sqcup$ and the least element $\perp \in \mathcal{S}$, and $(\mathcal{S}, \prec)$ is a strict partial order (i.e. antireflexive and transitive), linked together by the following axioms:

(1) $\forall A, B, C \in \mathcal{S}$, $B \sqsubseteq A$ and $A \prec C$ implies $B \prec C$.
(2) $\forall A, B, C \in \mathcal{S}$, $B \sqsubseteq A$ and $C \prec A$ implies $C \prec B$.
(3) $\forall A, B, C \in \mathcal{S}$, $A \prec C$ and $B \prec C$ implies $A \sqcup B \prec C$.
(4) $\forall A, B \in \mathcal{S}$, there exits $B_A \in \mathcal{S}$, called *cutting of A by B*, such that
  a) $B_A \prec A$ and $B_A \sqsubseteq B$;
  b) If $C \in \mathcal{S}$, $C \prec A$ and $C \sqsubseteq B$ then $C \sqsubseteq B_A$.

These axioms are schematically illustrated by the picture, which is for simplicity base on the causality structure of the Minkowski spacetime:

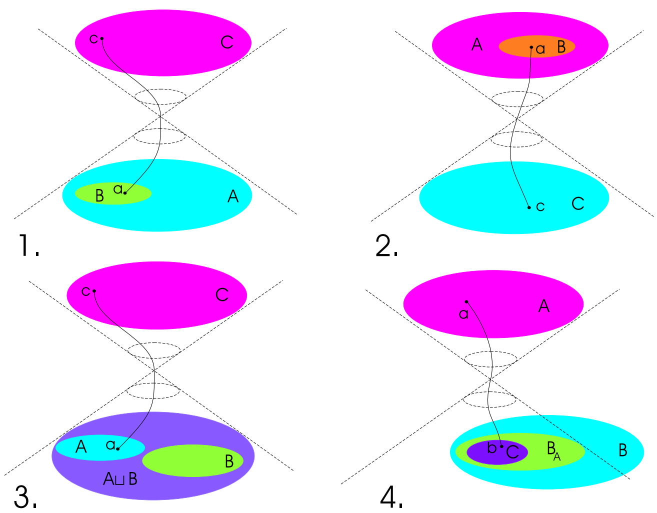

Figure 1. Crane-Christensen causal site.

The causal sites are natural generalization of *causal sets*, introduced and studied by R. Sorkin [11] and their study is motivated especially by the latest research in quantum gravity. The causal sites capture well the causality relationships in most

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models (including discrete models) of the spacetime, but the reconstruction of the topology of the spacetime from causality sites is a difficult open problem.

We will discuss the possibility of the reconstruction of the topological structure of the space time via the framework theory, recently developed by the author (elements of the framework theory had been discussed during the author's oral communication at the last Mathematical Workshop '05). Recall that a *framework* is a pair of sets (P, π), where $\pi \subseteq 2^P$. The elements of P are called *places*, the family π contains sets of those places which are "glued" together by possible presence of some physical object (e.g. a particle in some model of the reality) or simply by possible presence of a "virtual observer". Elements of π are also called *abstract points*. Finite frameworks may be very illustratively represented by the pin-stripe model, where stripes represent the places and pins stand for the abstract points:

Figure 2. The pin-stripe model.

The framework structure allows very flexible modeling of various topological relationships and a natural duality construction allows to switch easily between the pointless and the traditional point-set approach to the reality. The concept of a framework has some common background with FCA, formal concept analysis, which was founded by B. Ganter and R. Wille, now a successful theory widely applied in data analysis and artificial inteligence. Thus it bridges three relatively independent branches of the science: topology, computer science and theoretical physics and promises some foundation for more close cooperation of these scientific disciplines.

References

[1] Adámek, *Theory of mathematical structures*, D. Reidel Publishing Company, Dordrecht, 1983, pp. 1-317.
[2] Adámek J., Herrlich H., Strecker G. E., *Abstract and Concrete Categories – The Joy of Cats*, SNTL Praha, 2004, pp. 1-524.
[3] Adámek J., Koubek V., Reiterman J., *Foundations of general topology*, SNTL Praha, 1977, pp. 1-280 (Czech).
[4] Chistensen J. D., Crane L., *Causal Sites as Quantum Geometry*, preprint (2005), 1-21.
[5] Crane L., *Categorial Geometry as the Mathematical Foundations of Quantum Gravity*, preprint (2005), 1-15.
[6] Crane L., *Relational Topology as the Foundation of Quantum Gravity*, MPLA (World Scientific) 20 (2005), 2017-2035.

[7] Feynman R. P., Leighton R. B., Sands M., *The Feynman lectures on physics*, Addison Wesley Longman 1/3, 1977, pp. 1-732.
[8] Kopperman R., Wilson R., *Finite approximation of compact Hausdorff spaces*, preprint (2004), 1-14.
[9] Rideout D. P., *Dynamics of Causal Sets*, dissertation, Syracuse University, 2001, pp. 1-126.
[10] Schiller C., *Motion Mountain*, 2006, pp. 1-1366.
[11] Sorkin S., *Causal sets: Discrete gravity*, preprint (2003), 1-27.

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