A note on fuzzy preference structures

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1 Introduction to preference structures and fuzzy preference structures

The preference structure is a basic step of preference modeling. Given two alternatives, decision maker defines three binary relation-preference, indifference and incomparability.

A preference structure is a basic concept of preference modelling. In a classical preference structure (PS) a decision-maker makes three decission for any par (a, b) from the set $\boldsymbol{A}$ of all alternatives. His decision define a triplet P, I, J of a crisp binary relations on $\boldsymbol{A}$:

1) a is prefered to $b \Leftrightarrow (a, b) \in P$ (strict preference).
2) a and b are indifferent $\Leftrightarrow (a, b) \in I$ (indifference).
3) a and b are incomparable $\Leftrightarrow (a, b) \in J$ (incomparability).

A preference structure (PS) on a set $\boldsymbol{A}$ is a triplet (P, I, J) of binary relations on $\boldsymbol{A}$ such that

(ps1) I is reflexive, P and J are irreflexive.

(ps2) P is asymmetric, I and J are symmetric.

(ps3) $P \cap I = P \cap J = I \cap J = \emptyset$.

(ps4) $P \cup I \cup J \cup P^t = A \times A$ where $P^t(x, y) = P(y, x)$.

A preference structure can be characterized by the reflexive relation $R = P \cup I$ called the large preference relation. The relation R can be interpreted as

$$(a, b) \in R \Leftrightarrow a \text{ is prefered to } b \text{ or } a \text{ and } b \text{ are indifferent.}$$

It can be easily proved that

$$co(R) = P^t \cup J$$

where $coR(a, b) = 1 - R(a, b)$ and

$$P = R \cap co(R^t), I = R \cap R^t, J = co(R) \cap co(R^t).$$

It allows us to construct a preference structure (P, I, J) from a reflexive binary operation R only.

Decision-makers are often uncertain even inconsistent in their judgements. The restriction on two-valued relations have been an important drawback to their practical use. A natural demand led researchers to the introduction of a fuzzy preference structure (FPS). The original idea of using numbers between zero and one to describe the strenth of links between two alternatives goes back to Menger. The introduction of fuzzy relations allowed to express degrees of preference, indifference and incomparability. Of course, the attempts simply to replace the notion used in the definition of (PS) by their fuzzy equivalents have met some problems.

To define (FPS) it is necessary to consider some fuzzy connectives. We shall consider a continuous De Morgan triple (T, S, N) consisting of a continuous t-norm T, continuous t-conorm S and a strong negator N such that $T(x, y) = N(S(N(x), N(y)))$. The main problem consists in the fact that the completeness condition (ps4) can be written in many forms, e.g.:

$$co(P \cup P^t) = I \cup J, P = co(P^t \cup I \cup J), P \cup I = co(P^t \cup J).$$

Let (T,S,N) be De Morgan triplet. A fuzzy preference structure (FPS) on a set $\boldsymbol{A}$ is a triplet (P, I, J) of binary fuzzy relations on $\boldsymbol{A}$ such that

(f1) I is reflexive, P and J are irreflexive. $I(a, a) = 1, P(a, a) = J(a, a) = 0$

(f2) P is T-asymmetrical, I and J are symmetrical. $T(P(a, b), P(b, a)) = 0$

(f3) $T(P, I) = T(P, J) = T(I, J) = 0$. for all pair of alternatives

(f4) $(\forall (a, b) \in A^2) S(P, P^t, I, J) = 1$ or $N(S(P, I)) = S(P^t, J)$ or another completeness conditions.

One practical example will be given.

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