On Modeling of Water Flow in Vaneless Machines

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Abstract

Model of water flow in vaneless machines with friction type boundary condition is discussed. Formulation in cylindrical coordinates and its discretization is proposed.

1 Introduction

Vaneless machines driven by power water or pressure air are used in various areas of technology, e.g. in rotating washing brush, drilling machines, small water power stations etc. Although the efficiency is not too high they are popular for their simplicity, reliability and low price.

Principle of the vaneless machines is the following. The rotor on a shaft in bearings is axially symmetric. The working medium flows around the rotor such that trajectories of its particles form screw curves. Due to viscosity the fluid yields part of its kinetic energy to the rotor. To increase the efficiency, the surface of the rotor is rough while the surface of static parts is smooth. There are few papers dealing with their modeling.

2 Setting of the model

We shall confine modeling to working space of the machine around conic surface of rotor. Since we deal with vaneless machines driven by power water the assumption of incompressible fluid flow is adopted. Due to symmetry of the vaneless machine the pressure of the flowing liquid cannot transfer its energy to the rotor in contrast to the classical motors with vanes. Thus model of the ideal inviscid fluid cannot be used and viscosity must be considered.

Motion of viscous fluid is modeled by the the Navier-Stokes system of equations. For viscous liquids on the wall the non-slip condition is usually prescribed. The condition requires zero diference between surface velocity of liquid $\mathbf{u}$ and given velocity $\mathbf{u}_b$ of the wall: $\mathbf{u} - \mathbf{u}_b = 0$. Nevertheless, such condition pays no attention to the roughness of the wall which plays important role in the vaneless machines and thus is not realistic.

To describe this situation the following friction type boundary It states that the tangent stress $\mathbf{T_t}$ of liquid acting on the wall is proportional to difference of the liquid velocity and velocity of the wall: $\mathbf{T_t} = -\mu_0 \cdot (\mathbf{u} - \mathbf{u}_b)$ This linear dependence can be generalized to nonlinear dependence with a continuous positive function $g(\xi)$

$$\mathbf{T_t} = -g(|\mathbf{u} - \mathbf{u}_b)|) \cdot (\mathbf{u} - \mathbf{u}_b)\,. \tag{1}$$

3 Friction condition

To verify that this condition is consistent with the Navier-Stokes equation the weak formulation and existence of the solution was studied in [3]. The Navier-Stokes system of equations for viscous incompressible liquid, see e.g. [1], consists of continuity equation and vector balance equation. In the space of divergence free functions the weak formulation leads to the integral identity of the form

$$(\mathbf{u}_t, \mathbf{v}) + a(\mathbf{u}, \mathbf{v}) + b(\mathbf{u}, \mathbf{u}, \mathbf{v}) + \langle G(\mathbf{u} - \mathbf{u}_b), \mathbf{v} \rangle = (\mathbf{f}, \mathbf{v}) \,,$$

with the vector test function $\mathbf{v}$, viscous quadratic term a and cubic convection term b. The friction type condition (1) contributes with the quadratic term with G. Since it has "correct" sign, using convenient cut-off function coercivity of the operator can be achieved. Since the operators are weakly continuous the existence follows, see [2].

4 Problem in cylindrical coordinations and discretization

In the cylindrical coordinates (r, z, φ) the axially symmetric problem on studied volume Ω of the form the space between two concentric cone surfaces transforms to the system of equations on a trapezoidal domain G in the (r, z)-plane. Denoting the physical components of velocity vector (v_r, v_z, v_φ) by (u, v, w) and pressure by p the equations can be written in the form

$$(ru)_r + (rv)_z = 0 \,,$$

$$(r\,u)_t + \left(r\,u^2\right)_r + (r\,u\,v)_z + (rp)_r - w^2 = \nu \left[(r\,u_r)_r + (r\,u_z)_z - \frac{u}{r} \right] \,,$$

$$(r\,v)_t + (r\,v\,u)_r + \left(r\,v^2\right)_z + (rp)_z = \nu \left[(r\,v_r)_r + (r\,v_z)_z \right] \,,$$

$$(r\,w)_t + (r\,w\,u)_r + (r\,w\,v)_z + w\,u = \nu \left[(r\,w_r)_r + (r\,w_z)_z - \frac{w}{r} \right] \,.$$

On the input part of the boundary $z = z_{in}$ the velocity is prescribed, the output condition is prescribed on the line $z = z_{out}$. On the lines corresponding to rotor and stator surface the condition of type (1) is required.

The equations are written in conservative form whenever if was possible. For finite volume discretizaton the trapezoidal domain is decomposed by two systems of lines into $n \times m$ trapezoidal finite volumes. The equations are discretized by means of finite volume method with staggered grid proposed by Patankar in [4].

5 Conclusions

The initial boundary value problem for system of PDE completed with boundary conditions is formulated. Introduced friction type condition is consistent with the equation system. The problem is transformed into cylindrical coordinates and discretized by finite volume method. Next step will be testing the problem by numerical experiments and its comparison with the measurements in real machines.

References

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