S-map and its dual function

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We have more than one ways for modelling of joint distributions while studying some problems of vectors of random variables. Our approach is based on a generalized algebraic models - orthomodular lattices. We will study a dual function to s-map, which will be called j-map. Indeed, on any Boolean algebra s-map is a measure of intersection and j-map is a measure of uniom. We show some examples of such non Boolean models.

Definition 1. 1 *Let L be a nonempty set endowed with a partial ordering $\leq$. Let the greatest element (I) and the smallest element (O) exist and let the operations supremum $(\vee)$, infimum $\wedge$ (the lattice operations) be defined. Let $\perp: L \to L$ be a map with the following properties:*

(i) $\forall a, b \in L$, $a \vee b, a \wedge b \in L$.

(ii) $\forall a \in L\ \exists! a^{\perp} \in L$ *such that* $(a^{\perp})^{\perp} = a$ *and* $a \vee a^{\perp} = I$.

(iii) If $a, b \in L$ *and* $a \leq b$, *then* $b^{\perp} \leq a^{\perp}$.

(iv) If $a, b \in L$ *and if* $a \leq b$, *then* $b = a \vee (a^{\perp} \wedge b)$ *(orthomodular law).*

Then $(L, O, I, \vee, \wedge, \perp)$ *is called an orthomodular lattice (briefly an OML).*

Definition 1. 2 *Let L be a OML. The map $p : L^2 \to [0, 1]$ will be called s-map if the following conditions hold:*

(s1) $p(I, I) = 1$;

(s2) if $a \perp b$, *then* $p(a, b) = 0$ *and for* $\forall c \in L$

$$p(a \vee b, c) = p(a, c) + p(b, c)$$

$$p(c, a \vee b) = p(c, a) + p(c, b)$$

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Definition 1. 3 *Let L be a OML. A map $q : L \times L \to [0.1]$ will be called a join map (j-map) if the following conditions hold:*

(q1) $q(O, O) = 0$ *and* $q(I, I) = 1$;

(q2) If $a, b \in L$ *and* $a \perp b$, *then* $q(a, b) = q(a, a) + q(b, b)$;

(q3) If $a, b \in L$, *then for each* $c \in L$

$$q(a \vee b, c) = q(a, c) + q(b, c) - q(c, c)$$

$$q(c, a \vee b) = q(c, a) + q(c, b) - q(c, c)$$

Proposition 1. 1 *Let L be an OML and let q is j-map. Then $\mu(a) = q(a, a)$ is a state on L.*

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