ESTIMATIONS OF SOLUTIONS OF HYBRID DIFFERENTIAL SYSTEMS

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Abstract

Results on estimation of solutions of hybrid systems on finite time intervals are formulated as a result of our investigation. We are concerned with hybrid differential systems in the supposition that on every partial time interval it is represented by a linear differential system with constant coefficients.

We investigate hybrid differential systems on finite time intervals in the case when on every partial time interval it is represented by a linear differential system with constant coefficients. The main goal is to obtain inequalities for solutions of initial problems on given time interval. As a tool of investigation, method of Lyapunov functions is used.

Let a hybrid logic-dynamical system is expressed by subsystems

$$\dot{x}(t) = A_i x(t), \quad i = 1, \ldots, N, \quad x \in \mathbb{R}^n, \quad t_{i-1} \leq t < t_i, \quad t_0 = 0 \tag{1}$$

1. Application of square Lyapunov functions.

The following result is obtained with the aid of a Lyapunov function having a quadratic form.

Theorem 1 *Let the initial state of logic-dynamical system satisfies inequality*

$$|x(0)| < \delta.$$

Then

$$|x(t_N)| < \frac{\delta}{\sqrt{\prod_{i=1}^{N} \lambda_{min}[H_i(t_i)]}} \tag{2}$$

with

$$H_i(t_i) = e^{-A_i^T (t_i - t_{i-1})} e^{-A_i (t_i - t_{i-1})}, \quad i = 1, \ldots, N, \quad t_0 = 0.$$

2. Application of interval Lyapunov functions coinciding at boundary points.

We considered the case, when level surfaces given by a Lyapunov function on a preceding interval is an ellipse coinciding at the corresponding knot with an ellipse defined by a Lyapunov function used on the following interval.

Theorem 2 *Let the initial state of logic-dynamical system satisfies inequality*

$$|x(0)| < \delta.$$

Then

$$|x(t_N)| < \frac{\delta}{\sqrt{\lambda_{min}[H_N(t_N)]}} \tag{3}$$

with

$$H_N(t_N) = \prod_{i=N}^{1} e^{-A_i^T (t_i - t_{i-1})} \prod_{j=1}^{N} e^{-A_j (t_j - t_{j-1})}, \quad i = 1, \ldots, N, \quad t_0 = 0.$$