Mathematical modelling in radiobiology

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Radiobiology is dealing with study of the influence of ionizing radiation to the living organisms. Its results can find use in various areas of human life from which we can name e.g. radiotherapy - treatment of cancer by irradiating the tumor cells.

Although we know a lot about the effect of the ionizing radiation to the organism from various experiments, out of which some were carried out more thana hundred years ago, the are aspects of the radiobiological mechanism which are still not entirely understood.

To uncover these "secrets" it would be beneficial to have a mathematical model of the whole radiobiological process. In the talk we shall see how to model a little piece of this very complex problem.

The key reason for the cell's death caused by irradiation is the damage of the DNA. In such case the cell does not have to die immediately, however such a cell is often unable to produce some type of enzyme necessary for further life. Observing the impact of the irradiation to the DNA we can divide the processes taking place in an irradiated tissue into the following four phases:

(i) A particle passing through the cell's environment transfers its kinetic energy to the surrounding aqueous solution which causes dissociation of the water molecules and creation of aggressive radicals.

(ii) The radicals and ions generated in the previous phase diffuse and react with each other and with possible interaction with the DNA which with a certain probability can harm it.

(iii) The cell starts a set of enzymatic reactions trying to repair the given damage. This phase results in either complete recovery of the cell or its inactivation (i.e. death).

(iv) The whole tissue or organism reacts to the given damage.

In the talk we shall concentrate on the the second, the so-called physio-chemical phase. The problem of diffusion and recombination of radicals can be described by a system of non-linear parabolic partial differential equations

$$(1) \quad \frac{\partial c_i}{\partial t}(x,t) = D_i \triangle_x c_i(x,t) - \sum_{j \neq i} k_{ij} c_i(x,t) c_j(x,t) + \sum_{j,k \neq i} k_{jk} c_j(x,t) c_k(x,t), \; i = 1, ..., n,$$

for the concentrations of each kind of radicals involved in the process.

Here x is a space variable in an unbounded three dimensional domain and certain multiple of the Dirac delta function is taken as an initial condition.

We shall outline a proof of the spherical symmetry of the solution to the system (1) which we shall use in simplifying the system to one dimensional form

$$(2)\quad \frac{\partial C_i}{\partial t}(r,t) = D_i \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C_i}{\partial r} \right) \right] - \sum_{j \neq i} k_{ij} C_i(r,t) C_j(r,t) + \sum_{\{j,k\} \in \mathcal{M}_i} k_{jk} C_j(r,t) C_k(r,t)$$

We also note how to make a connection between the solution of the system (2) and the probability of the DNA damage.

Finally we note how to deal with the singular initial condition and propose a numerical method for solving this system