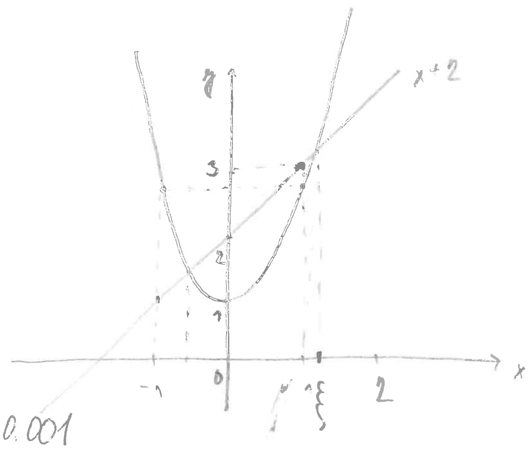
① BISEKCE:

$$e^{x^2} - x - 2 = 0$$



a) separace kořenů: $e^{x^2} = x + 2$

kl. kořen: $\xi \in [1,2]$

$f(1) = e - 1 - 2 < 0 \quad (-0.2817)$

$f(2) = e^4 - 4 > 0 \quad (50.59815)$

b) odhad počtu kroků: $a_0 = 1,\ b_0 = 2,\ \varepsilon = 0.001$

$$\frac{b_0 - a_0}{2^{n+1}} < \varepsilon$$

$$\frac{1}{2^{n+1}} < \frac{1}{1000}$$

$$1000 < 2^{n+1} \qquad / \ln_2(\ )$$

$$\ln_2(1000) < n+1$$

$$n > 9.9658 - 1$$

$$\underline{\underline{n = 9}}$$

c) iterační proces

i	a_i	b_i	s_i	$f(a_i)$	$f(b_i)$	$f(s_i)$	d_i
0	1	2	1,5	-0.2817	50.5982	5.9877	0.5
1	1	1.5	1,25	-0.2817	5.9877	1.5207	0,25
2	1	1,25	1.125	-0.2817	1.5207	0.4203	0,125

$\hat{x} = 1.125 \pm 0,125$

② • GEM bez výměny řádků: LU rozklad + řešení

NELZE!

$$x_1 - x_2 + 2x_3 - x_4 = -8$$
$$2x_1 - 2x_2 + 3x_3 - 3x_4 = -20$$
$$x_1 + x_2 + x_3 = -2$$
$$x_1 - x_2 + 4x_3 + 3x_4 = 4$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -8 \\ 2 & -2 & 3 & -3 & -20 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 4 \end{array}\right) \begin{array}{l} \\ II - 2\cdot I \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -8 \\ 0 & \underline{\underline{0}} & & & \\ & & & & \\ & & & & \end{array}\right)$$

... nelze bez výměny řádků

• GEM s částečným výběrem pivota

$$\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix} \to \begin{pmatrix} 0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix} \to \begin{pmatrix} 0&1&0&0\\0&0&1&0\\1&0&0&0\\0&0&0&1 \end{pmatrix} \to \begin{pmatrix} 0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0 \end{pmatrix} = P$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -8 \\ 2 & -2 & 3 & -3 & -20 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 1 & -1 & 2 & -1 & -8 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 4 \end{array}\right) \begin{array}{l} \\ II - \frac{1}{2}\cdot I \\ III - \frac{1}{2}\cdot I \\ IV - \frac{1}{2}\cdot I \end{array} \sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} & 8 \\ 0 & 0 & \frac{5}{2} & \frac{9}{2} & 14 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} & 8 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 0 & \frac{5}{2} & \frac{9}{2} & 14 \end{array}\right) \sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} & 8 \\ 0 & 0 & \frac{5}{2} & \frac{9}{2} & 14 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 2 \end{array}\right) \begin{array}{l} \\ \\ \\ IV - \frac{1}{5} III \end{array} \sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} & 8 \\ 0 & 0 & \frac{5}{2} & \frac{9}{2} & 14 \\ 0 & 0 & 0 & -\frac{2}{5} & -\frac{4}{5} \end{array}\right)$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{2} & 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{5} & 1 \end{pmatrix}; \quad U = \begin{pmatrix} 2 & -2 & 3 & -3 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & \frac{5}{2} & \frac{9}{2} \\ 0 & 0 & 0 & -\frac{2}{5} \end{pmatrix}; \quad P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \quad P\cdot b = \begin{pmatrix} -20 \\ -2 \\ 4 \\ -8 \end{pmatrix}$$

$\boxed{1.\ Ly = Pb}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{2} & 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} -20 \\ -2 \\ 4 \\ -8 \end{pmatrix}$$

$$y_1 = -20$$
$$\frac{1}{2}y_1 + y_2 = -2 \Rightarrow y_2 = 8$$
$$\frac{1}{2}y_1 + y_3 = 4 \Rightarrow y_3 = 14$$
$$\frac{1}{2}y_1 + \frac{1}{5}y_3 + y_4 = -8 \Rightarrow y_4 = -\frac{4}{5}$$

$$\Rightarrow y = \begin{pmatrix} -20 \\ 8 \\ 14 \\ -4/5 \end{pmatrix}$$

$$-10 + \frac{14}{5} = -\frac{36}{5}$$
$$-8 + \frac{36}{5} = -\frac{4}{5}$$

$\boxed{2.\ Ux = y}$

$$\begin{pmatrix} 2 & -2 & 3 & -3 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & \frac{5}{2} & \frac{9}{2} \\ 0 & 0 & 0 & -\frac{2}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -20 \\ 8 \\ 14 \\ -4/5 \end{pmatrix}$$

$$2x_1 - 2x_2 + 3x_3 - 3x_4 = -20 \Rightarrow x_1 = -4$$
$$2x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 = 8 \Rightarrow x_2 = 3$$
$$\frac{5}{2}x_3 + \frac{9}{2}x_4 = 14 \Rightarrow x_3 = 2$$
$$-\frac{2}{5}x_4 = -4/5 \Rightarrow x_4 = 2$$

$$\Rightarrow x = \begin{pmatrix} -4 \\ 3 \\ 2 \\ 2 \end{pmatrix}$$

③ GEM bez výměny řádků : LU rozklad

$$
\begin{aligned}
-x_1 \phantom{{}-x_2} + 3x_3 + 2x_4 &= -1 \\
5x_1 - x_2 + 3x_3 + 2x_4 &= 8 \\
x_1 - x_2 + 2x_3 - 2x_4 &= 4 \\
4x_2 + x_3 - 2x_4 &= -4
\end{aligned}
$$

$$
\left(\begin{array}{cccc|c}
-1 & 0 & 3 & 2 & -1 \\
5 & -1 & 3 & 2 & 8 \\
1 & -1 & 2 & -2 & 4 \\
0 & 4 & 1 & -2 & -4
\end{array}\right)
\begin{array}{l}
\\
II-(-5)\cdot I \\
III-(-1)\cdot I \\
IV-(0)\cdot I
\end{array}
\sim
\left(\begin{array}{cccc|c}
-1 & 0 & 3 & 2 & -1 \\
0 & -1 & 18 & 12 & 3 \\
0 & -1 & 5 & 0 & 3 \\
0 & 4 & 1 & -2 & -4
\end{array}\right)
\begin{array}{l}
\\
\\
III-1\cdot II \\
IV-(-4)\cdot II
\end{array}
\sim
$$

$$
\sim
\left(\begin{array}{cccc|c}
-1 & 0 & 3 & 2 & -1 \\
0 & -1 & 18 & 12 & 3 \\
0 & 0 & -13 & -12 & 0 \\
0 & 0 & 73 & 46 & 8
\end{array}\right)
\begin{array}{l}
\\
\\
\\
IV-\left(-\frac{73}{13}\right)\cdot III
\end{array}
\sim
\left(\begin{array}{cccc|c}
-1 & 0 & 3 & 2 & -1 \\
0 & -1 & 18 & 12 & 3 \\
0 & 0 & -13 & -12 & 0 \\
0 & 0 & 0 & -\frac{278}{13} & 8
\end{array}\right)
$$

$$
L = \begin{pmatrix}
1 & 0 & 0 & 0 \\
-5 & 1 & 0 & 0 \\
-1 & 1 & 1 & 0 \\
0 & -4 & -\frac{73}{13} & 1
\end{pmatrix};
\quad
U = \begin{pmatrix}
-1 & 0 & 3 & 2 \\
0 & -1 & 18 & 12 \\
0 & 0 & -13 & -12 \\
0 & 0 & 0 & \frac{-278}{13}
\end{pmatrix}
$$

4 Choleského rozklad + řešení

$$\begin{aligned} 4x_1 + 3x_2 &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= -24 \end{aligned}$$

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$$

- A sym. ✓
- $\left.\begin{aligned} |A_1| &= 4 > 0 \\ |A_2| &= 16 - 9 = 7 > 0 \\ |A_3| &= 64 - 4 - 36 = 24 > 0 \end{aligned}\right\}$ poz. def. ✓

- rozklad: $U^T U = A$

$$\begin{pmatrix} u_{11} & 0 & 0 \\ u_{12} & u_{22} & 0 \\ u_{13} & u_{23} & u_{33} \end{pmatrix} \cdot \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$$

$4 = u_{11}^2 \Rightarrow \boxed{u_{11} = 2}$

$3 = u_{11} \cdot u_{12} \Rightarrow \boxed{u_{12} = \frac{3}{2}}$

$0 = u_{11} \cdot u_{13} \Rightarrow \boxed{u_{13} = 0}$

$$U = \begin{pmatrix} 2 & \frac{3}{2} & 0 \\ 0 & \frac{\sqrt{7}}{2} & -\frac{2}{\sqrt{7}} \\ 0 & 0 & 2\sqrt{\frac{6}{7}} \end{pmatrix}$$

$4 = u_{12}^2 + u_{22}^2 \Rightarrow \boxed{u_{22}} = \sqrt{4 - \frac{9}{4}} = \sqrt{\frac{7}{4}} = \boxed{\frac{\sqrt{7}}{2}}$

$-1 = u_{12} \cdot u_{13} + u_{22} \cdot u_{23} \Rightarrow \boxed{u_{23}} = \dfrac{-1 - \frac{3}{2} \cdot 0}{\frac{\sqrt{7}}{2}} = \boxed{-\frac{2}{\sqrt{7}}}$

$4 = u_{13}^2 + u_{23}^2 + u_{33}^2 \Rightarrow \boxed{u_{33}} = \sqrt{4 - 0^2 - \left(-\frac{2}{\sqrt{7}}\right)^2} = \sqrt{4 - \frac{4}{7}} = \sqrt{\frac{24}{7}} = \sqrt{\frac{4 \cdot 6}{7}} = \boxed{2\sqrt{\frac{6}{7}}}$

- řešení: $U^T \underbrace{U x}_{y} = b$

$\boxed{1.\ U^T y = b}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 24 \\ \frac{3}{2} & \frac{\sqrt{7}}{2} & 0 & 30 \\ 0 & -\frac{2}{\sqrt{7}} & 2\sqrt{\frac{6}{7}} & -24 \end{array}\right)$$

$2y_1 = 24 \Rightarrow \underline{y_1 = 12}$

$\frac{3}{2} y_1 + \frac{\sqrt{7}}{2} y_2 = 30 \Rightarrow \underline{y_2} = \frac{2}{\sqrt{7}}(30 - 18) = \underline{\frac{24}{\sqrt{7}}}$

$-\frac{2}{\sqrt{7}} y_2 + 2\sqrt{\frac{6}{7}} y_3 = -24 \Rightarrow \underline{y_3} = \frac{1}{2\sqrt{\frac{6}{7}}}\left(-24 + \frac{48}{7}\right) = \frac{\sqrt{7}}{2\sqrt{6}} \cdot \frac{-120}{7} = \underline{\frac{-60}{\sqrt{42}}}$

$$y = \begin{pmatrix} 12 \\ 24/\sqrt{7} \\ -60/\sqrt{42} \end{pmatrix}$$

$\boxed{2.\ U x = y}$

$$\left(\begin{array}{ccc|c} 2 & \frac{3}{2} & 0 & 12 \\ 0 & \frac{\sqrt{7}}{2} & -\frac{2}{\sqrt{7}} & \frac{24}{\sqrt{7}} \\ 0 & 0 & 2\sqrt{\frac{6}{7}} & -\frac{60}{\sqrt{42}} \end{array}\right)$$

$2x_1 + \frac{3}{2} x_2 = 12 \Rightarrow \underline{x_1} = \frac{1}{2}(12 - 6) = \underline{3}$

$\frac{\sqrt{7}}{2} x_2 - \frac{2}{\sqrt{7}} x_3 = \frac{24}{\sqrt{7}} \Rightarrow \underline{x_2} = \frac{2}{\sqrt{7}}\left(\frac{24}{\sqrt{7}} - \frac{10}{\sqrt{7}}\right) = \frac{28}{7} = \underline{4}$

$2\sqrt{\frac{6}{7}} x_3 = -\frac{60}{\sqrt{42}} \Rightarrow \underline{x_3} = \frac{-30}{\sqrt{42}} \cdot \sqrt{\frac{7}{6}} = \frac{-30}{\sqrt{36}} = \underline{-5}$

$$x = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

5. Newtonův interpolační polynom:

čas [s]	1	3	5	7	13
rychlost [cm·s^{-1}]	800	2310	3090	3940	4755

x	y	$y(x_i, x_{i+1})$	$y(x_i, x_{i+1}, x_{i+2})$	$y(x_i, \dots, x_{i+3})$	$y(x_i, x_{\dots}, x_{i+4})$
1	800	$\frac{1510}{2} = 755$	$\frac{-365}{4}$	$\frac{166\,667}{10\,000}$	$-\frac{1463}{1000}$
3	2310	$\frac{780}{2} = 390$	$\frac{35}{4}$	$-\frac{2\,806}{625}$	
5	3090	$\frac{850}{2} = 425$	$-\frac{\frac{1735}{6}}{8} = -\frac{1735}{48}$		
7	3940	$\frac{815}{6}$			
13	4755				

$$N(x) = 800 + 755\underset{9}{(x-1)} - \frac{365}{4}\underset{9}{(x-1)}\underset{7}{(x-3)} + \frac{166\,664}{10\,000}\underset{9}{(x-1)}\underset{7}{(x-3)}\underset{5}{(x-5)} - \frac{1763}{1000}\underset{9}{(x-1)}\underset{7}{(x-3)}\underset{5}{(x-5)}\underset{3}{(x-7)}$$

$$N(10) = \frac{48\,872}{9} = \underline{\underline{5\,430{,}2 \text{ cm}\cdot\text{s}^{-1}}}$$

6. Naměřené hodnoty y_i v uzlech x_i

x_i	-2	-1	1	2
y_i	-3	2	0	1

proložte lin. kombinací fcí 1, x^1 a x^2 pomocí MNČ.

→ Choleského rozklad

→ [illegible] odhad v b. 0

$$\varphi^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \varphi^{(2)} = \begin{pmatrix} -2 \\ -1 \\ 1 \\ 2 \end{pmatrix}, \quad \varphi^{(3)} = \begin{pmatrix} 4 \\ 1 \\ 1 \\ 4 \end{pmatrix}, \quad \varphi = \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$\langle \varphi^{(1)}, \varphi^{(1)} \rangle = 4$

$\langle \varphi^{(1)}, \varphi^{(2)} \rangle = 0$

$\langle \varphi^{(1)}, \varphi^{(3)} \rangle = 10$

$\langle \varphi^{(2)}, \varphi^{(2)} \rangle = 10$

$\langle \varphi^{(2)}, \varphi^{(3)} \rangle = 0$

$\langle \varphi^{(3)}, \varphi^{(3)} \rangle = 34$

$\langle \varphi, \varphi^{(1)} \rangle = 0$

$\langle \varphi, \varphi^{(2)} \rangle = 6$

$\langle \varphi, \varphi^{(3)} \rangle = -6$

$$\Rightarrow \begin{pmatrix} 4 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 34 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ -6 \end{pmatrix}$$

• Choleského rozklad:

$$\left(\begin{array}{ccc|c} 4 & 0 & 10 & 0 \\ 0 & 10 & 0 & 6 \\ 10 & 0 & 34 & -6 \end{array}\right)$$

• A sym. ✓

• $|A_1| = 4 > 0$

$|A_2| = 40 > 0$

$|A_3| = 1360 - 1000 = 360 > 0$

$\Big\}$ poz. def. ✓

$$\begin{pmatrix} u_{11} & 0 & 0 \\ u_{12} & u_{22} & 0 \\ u_{13} & u_{23} & u_{33} \end{pmatrix} \cdot \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} 4 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 34 \end{pmatrix}$$

$4 = u_{11}^2 \Rightarrow \boxed{u_{11} = 2}$

$0 = u_{11} \cdot u_{12} \Rightarrow \boxed{u_{12} = 0}$

$10 = u_{11} \cdot u_{13} \Rightarrow \boxed{u_{13} = \frac{10}{2} = 5}$

$10 = u_{12}^2 + u_{22}^2 \Rightarrow \boxed{u_{22} = \sqrt{10 - 0} = \sqrt{10}}$

$0 = u_{12} \cdot u_{13} + u_{22} \cdot u_{23} \Rightarrow \boxed{u_{23} = 0}$

$34 = u_{13}^2 + u_{23}^2 + u_{33}^2 \Rightarrow \boxed{u_{33} = \sqrt{34 - 25} = 3}$

$$\Rightarrow U = \begin{pmatrix} 2 & 0 & 5 \\ 0 & \sqrt{10} & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$U^T \underbrace{U \mathbf{x}}_{\mathbf{y}} = \mathbf{b}$

$\boxed{1.\ U^T \mathbf{y} = \mathbf{b}}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & \sqrt{10} & 0 & 6 \\ 5 & 0 & 3 & -6 \end{array}\right)$$

$2y_1 = 0 \Rightarrow y_1 = 0$

$\sqrt{10}\, y_2 = 6 \Rightarrow y_2 = \frac{6}{\sqrt{10}}$

$5y_1 + 3y_3 = -6 \Rightarrow y_3 = \frac{-6}{3} = -2$

$$\Rightarrow \mathbf{y} = \begin{pmatrix} 0 \\ \frac{6}{\sqrt{10}} \\ -2 \end{pmatrix}$$

$\boxed{2.\ U\mathbf{x} = \mathbf{y}}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 5 & 0 \\ 0 & \sqrt{10} & 0 & \frac{6}{\sqrt{10}} \\ 0 & 0 & 3 & -2 \end{array}\right)$$

$2x_1 + 5x_3 = 0 \Rightarrow x_1 = \frac{1}{2} \cdot \frac{10}{3} = \frac{5}{3}$

$\sqrt{10}\, x_2 = \frac{6}{\sqrt{10}} \Rightarrow x_2 = \frac{6}{10} = \frac{3}{5}$

$3x_3 = -2 \Rightarrow x_3 = -\frac{2}{3}$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 5/3 \\ 3/5 \\ -2/3 \end{pmatrix}$$

$\Rightarrow$ vracím se k MNČ:

$\mathbf{c} = \left(\frac{5}{3}, \frac{3}{5}, -\frac{2}{3}\right)^T \Rightarrow \varphi(x) = \frac{5}{3} + \frac{3}{5}x - \frac{2}{3}x^2$

$$\varphi^* = \frac{5}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} -2 \\ -1 \\ 1 \\ 2 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} 4 \\ 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -11/5 \\ 2/5 \\ 8/5 \\ 1/5 \end{pmatrix}$$

$$\varphi - \varphi^* = \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -11/5 \\ 2/5 \\ 8/5 \\ 1/5 \end{pmatrix} = \begin{pmatrix} -4/5 \\ 8/5 \\ -8/5 \\ 4/5 \end{pmatrix}$$

$\|\varphi - \varphi^*\|_2 = \frac{421}{285}$

$\varphi(0) = \frac{5}{3}$