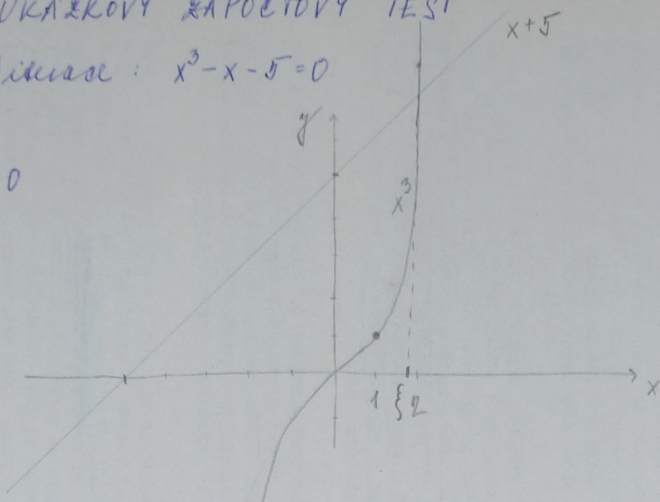
CA001 – UKÁZKOVÝ ZÁPOČTOVÝ TEST

① Metoda prosté iterace: $x^3 - x - 5 = 0$

$g(x) = \sqrt[3]{x+5}$

Řešení! $x^3 - x - 5 = 0$

$x^3 = x + 5$



$f(x) = x^3 - x - 5$

$\left.\begin{array}{l} f(1) = -5 \\ f(2) = 1 \end{array}\right\} f(1) \cdot f(2) = -5 < 0$

$\Downarrow$

$\boxed{I = [1,2]}$

• ověření podmínek: $g(x) = \sqrt[3]{x+5}$, $I = [1,2]$:

1. $g(x) \in C(I)$ ✓ (není žádný bod nespojitosti na I)
2. zobrazení do sebe: $g(1) = \sqrt[3]{6} \doteq 1.8171 \in I$

   $g(2) = \sqrt[3]{7} \doteq 1.9129 \in I$ ✓

   extrém? $f'(x) = \frac{1}{3}(x+5)^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{(x+5)^2}} = 0$ … nemá řešení $\Rightarrow$ nemá extrém na I

   $\Rightarrow$ na I je $g(x)$ monotonní

   $\Rightarrow g: I \to I$ ✓.

3. $g'(x) = \frac{1}{3\sqrt[3]{(x+5)^2}}$

   $|g'(1)| = \frac{1}{3 \cdot \sqrt[3]{6^2}} \doteq 0.1 < 1$

   $|g'(2)| = \frac{1}{3\sqrt[3]{7^2}} \doteq 0.0911 < 1$

   $g''(x) = \frac{1}{3} \cdot -\frac{2}{3}(x+5)^{-\frac{5}{3}} = 0$ … nemá řešení $\Rightarrow g'(x)$ nemá extrém na $I \Rightarrow g'(x)$ je na I monotonní $\Rightarrow |g'(x)| < 1 \ \forall x \in I$ ✓

• výpočet, např. $x_0 = 1$

i	x_i	x_{i+1}	$\lvert x_{i+1} - x_i \rvert$
0	1	1.8171	0.8171
1	1.8171	1.8961	0.079
2	1.8961	1.9034	0.0073

② Newtonův interpolační polynom, aprox. $e^{0.15}$, $x_0 = 0.1$, $x_1 = 0.2$

Řešení!

i	x_i	y_i	$y(x_0, x_1)$
0	0.1	1.1052	1.1623
1	0.2	1.2214	

$N(x) = 1.1052 + 1.1623(x - 0.1)$

$N(0.15) = 1.1052 + 1.1623(0.15 - 0.1) = 1.1633$

odhad abs. chyby: $|N(0.15) - e^{0.15}| = |1.1633 - 1.1618| = \underline{0.0015}$

③

$$4x_1 + 3x_2 = 24$$
$$3x_1 + 4x_2 - x_3 = 30$$
$$-x_2 + 4x_3 = -24$$

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$$

• A ryze řádkově diagonálně dominantní?

$|4| > |3|$ ✓

$|4| \geq |3| + |-1|$ (na testu případ, kdy nejsou splněny podmínky, nebude ☺)

$|4| > |-1|$ ✓

$$D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & -0 \\ 0 & -0 & 4 \end{pmatrix} \quad L = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \quad U = \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

G-S: $(D + L + U)x = b$

$(D + L)x^{(i+1)} = -Ux^{(i)} + b$

$$\begin{pmatrix} 4 & 0 & 0 \\ 3 & 4 & 0 \\ 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1^{(i+1)} \\ x_2^{(i+1)} \\ x_3^{(i+1)} \end{pmatrix} = \begin{pmatrix} 0 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1^{(i)} \\ x_2^{(i)} \\ x_3^{(i)} \end{pmatrix} + \begin{pmatrix} 24 \\ 30 \\ -24 \end{pmatrix} \rightarrow \begin{aligned} 4x_1^{(i+1)} &= -3x_2^{(i)} + 24 \\ 3x_1^{(i+1)} + 4x_2^{(i+1)} &= x_3^{(i)} + 30 \\ -x_2^{(i+1)} + 4x_3^{(i+1)} &= -24 \end{aligned}$$

$x^{(0)} = (0, 0, 0)^T$

$$x^{(1)}: \left.\begin{aligned} 4x_1^{(1)} = -3x_2^{(0)} + 24 &\Rightarrow x_1^{(1)} = 6 \\ 3x_1^{(1)} + 4x_2^{(1)} = x_3^{(0)} + 30 &\Rightarrow x_2^{(1)} = \frac{1}{4}(30 - 18) = 3 \\ -x_2^{(1)} + 4x_3^{(1)} = -24 &\Rightarrow x_3^{(1)} = \frac{1}{4}(-24 + 3) = -\frac{21}{4} \end{aligned}\right\} \Rightarrow \begin{aligned} x^{(1)} &= \left(6, 3, -\frac{21}{4}\right)^T \\ \|x^{(1)} - x^{(0)}\|_\infty &= 6 \end{aligned}$$

$$x^{(2)}: \left.\begin{aligned} 4x_1^{(2)} = -3x_2^{(1)} + 24 &\Rightarrow x_1^{(2)} = \frac{1}{4}(-9 + 24) = \frac{15}{4} \\ 3x_1^{(2)} + 4x_2^{(2)} = x_3^{(1)} + 30 &\Rightarrow x_2^{(2)} = \frac{1}{4}\left(-3 \cdot \frac{15}{4} - \frac{21}{4} + 30\right) = \frac{1}{4} \cdot \frac{\cancel{143}\,54}{4} = \frac{27}{8} \\ -x_2^{(2)} + 4x_3^{(2)} = -24 &\Rightarrow x_3^{(2)} = \frac{1}{4}\left(\frac{27}{8} - 24\right) = \frac{1}{4} \cdot \frac{-165}{8} = -\frac{165}{32} \end{aligned}\right\} \Rightarrow \begin{aligned} x^{(2)} &= \left(\frac{15}{4}, \frac{27}{8}, -\frac{165}{32}\right)^T \\ \|x^{(2)} - x^{(1)}\|_\infty &= 2.25 \end{aligned}$$