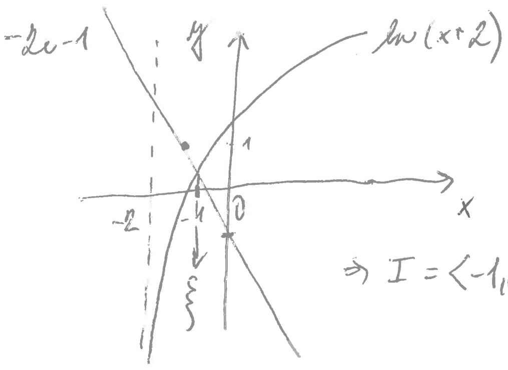
① Newtonova metoda:

- **SEPARACE KOŘENŮ**

$$\ln(x+2)+2x+1=0$$

$$\ln(x+2)=-2x-1$$



$\Rightarrow I=\langle -1,0\rangle$

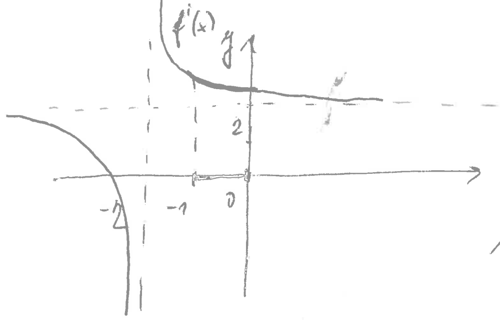
- $I=\langle -1,0\rangle$, podmínky konvergence:

1. $f(x)=\ln(x+2)+2x+1 \in C(I)$ ✓

$f'(x)=\dfrac{1}{x+2}+2 \in C(I)$ ✓

$f''(x)=-\dfrac{1}{(x+2)^2} \in C(I)$ ✓

2.



$\Rightarrow f'(x)>0$ na I

$\Downarrow$

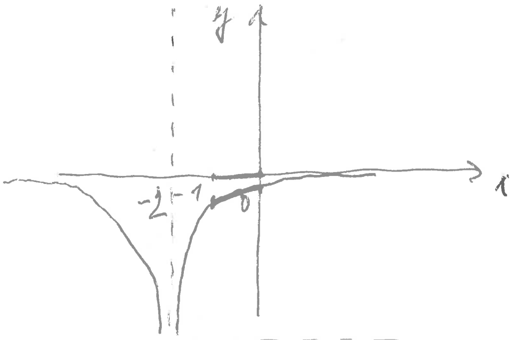
$f'(x)$ nemění na I znaménko ✓

3. volba x^0: $f(x^0)\,f''(x^0)>0$

$f(-1)=\ln 1-1=-1$

$f(0)=\ln 2+1>0$

$f''(x)<0 \;\forall x\in I \Rightarrow \boxed{x^0=-1}$



$\Rightarrow f''(x)<0$ na I

$\Downarrow$

$f''(x)$ nemění na I znaménko ✓

- **ITERAČNÍ PROCES:**

i	x^i	x^{i+1}	$\lvert x^{i+1}-x^i\rvert$
0	-1	$-2/3$	$1/3$
1	$-2/3$	-0.6501	0.0166

$-0.6501 \Downarrow x^2$

$0.0166 \Downarrow$ chyba

(2) Hermiteův interpolační polynom: $\cos(1{,}95)$

$f(x) = \cos x,\ x_0 = 1.9,\ x_1 = 2$

$\Downarrow$

$f'(x) = -\sin x$

i	0	1
x_i	1.9	2
$f(x_i)$	-0.3233	-0.4161
$f'(x_i)$	-0.9463	-0.9093

x_i	$f(x_i)$			
1.9	-0.3233			
		-0.9463		
1.9	-0.3233		0.183	
		-0.928		0.04
2	-0,4161		0.187	
		-0.9093		
2	-0.4161			

$$H(x) = -0.3233 - 0.9463(x-1{,}9) + 0{,}183(x-1{,}9)^2 + 0.04(x-1{,}9)^2(x-2)$$

$$\left.\begin{aligned} H(1.95) &= -0.3702 \\ \cos(1{,}95) &= -0{,}3702 \end{aligned}\right\} \ |H(1{,}95) - \cos(1{,}95)| < 10^{-4}$$

(3) Jacobiova metoda:

$$\begin{aligned} 4x_1 + 2x_2 + x_3 &= 1 \\ x_1 + 4x_2 + x_3 &= 5 \\ x_1 + x_2 + 3x_3 &= 0. \end{aligned}$$

$$A = \begin{pmatrix} 4 & 2 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

ŘDD (i ŘSDD) ✓

$$\boxed{\begin{aligned} 4x_1^{(i+1)} &= -2x_2^{(i)} - x_3^{(i)} + 1 \\ 4x_2^{(i+1)} &= -x_1^{(i)} - x_3^{(i)} + 5 \\ 3x_3^{(i+1)} &= -x_1^{(i)} - x_2^{(i)} \end{aligned}}$$

$x^{(0)} = (0,0,0)^T$

$$x^{(1)}: \left.\begin{aligned} 4x_1^{(1)} &= 1 \\ 4x_2^{(1)} &= 5 \\ 3x_3^{(1)} &= 0 \end{aligned}\right\} \Rightarrow x^{(1)} = \left(\frac{1}{4}, \frac{5}{4}, 0\right)^T; \quad \|x^{(1)} - x^{(0)}\|_\infty = \frac{5}{4}$$

$$x^{(2)}: \left.\begin{aligned} 4x_1^{(2)} &= -2x_2^{(1)} - x_3^{(1)} + 1 \\ 4x_2^{(2)} &= -x_1^{(1)} - x_3^{(1)} + 5 \\ 3x_3^{(2)} &= -x_1^{(1)} - x_2^{(1)} \end{aligned}\right\} \Rightarrow x^{(2)} = \left(-\frac{3}{8}, \frac{19}{16}, -\frac{1}{2}\right)^T; \quad \|x^{(2)} - x^{(1)}\|_\infty =$$