- **vzor1 - řešení:**

  1. a) $k = \frac{1}{15}$

     b) $EX = \frac{49}{15}$

     c) $P(X \geq 2) = \frac{14}{15}$, $P(X \in \{2,3\}) = \frac{2}{5}$

     d) $p(y) = \begin{cases} \frac{1}{15} 2^{4y-2} & y \in \left\{ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} \right\} \\ 0 & \text{jinak} \end{cases}$

  2. a) $F(x) = \begin{cases} 0 & x \leq 1 \\ \frac{5}{4} \left(1 - \frac{1}{x} \right) & x \in (1,5) \\ 1 & x \geq 5 \end{cases}$

     b) $P(X \leq 3) = \frac{5}{6}$, $P(2 < X < 4) = \frac{5}{16}$

     c) $EX = \frac{5}{4} \ln 5$, $EX^2 = 5$, $D(X) = 5 - \frac{25}{16} \ln^2 5$

  3. a) $X \sim p_1(x), p_1(x) = \begin{cases} \frac{1}{3} & x = 1 \\ \frac{2}{3} & x = 2 \\ 0 & \text{jinak}, \end{cases}$

     $Y \sim p_2(y), p_2(y) = \begin{cases} \frac{1}{7} & y = 0 \\ \frac{2}{7} & y = 1 \\ \frac{4}{7} & y = 2 \end{cases}$

     b) X a Y jsou stochasticky nezávislé

     c) $P(Y < 2) = \frac{3}{7}$, $P(X \geq 1, Y > 0) = \frac{6}{7}$

- **vzor2 - řešení:**

  1. a) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & 0 \leq x < 1 \\ \frac{3}{4} & 1 \leq x < 2 \\ \frac{7}{8} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$

     b) $P(X \leq 2) = \frac{7}{8}$

     c) $EX = \frac{7}{8}$, $EX^2 = \frac{15}{8}$, $DX = \frac{71}{64}$

  2. a) $F(x) = \begin{cases} 0 & x \leq 1 \\ -\frac{9}{8} \left(\frac{1}{x^2} - 1 \right) & x \in (1,3) \\ 1 & x \geq 3 \end{cases}$

     b) $P(X \leq 2) = \frac{27}{32}$, $P \left(\frac{3}{2} \leq X \leq \frac{5}{2} \right) = \frac{8}{25}$

     c) $g(y) = \begin{cases} \frac{9}{4y} \left(\ln y \right)^{-3} & y \in \left(e, e^3 \right) \\ 0 & \text{jinak} \end{cases}$

  3. a) $c = \frac{1}{12}$

     b) $X \sim f_1(x), f_1(x) = \begin{cases} \frac{1}{6}(x+2) & x \in (0,2) \\ 0 & \text{jinak} \end{cases}$

     $Y \sim f_2(y), f_2(y) = \begin{cases} \frac{1}{2}y & y \in (0,2) \\ 0 & \text{jinak} \end{cases}$

     X a Y jsou stochasticky nezávislé

     c) $P(X < 1) = \frac{5}{12}$, $P(X < 1, Y < 1) = \frac{5}{48}$

- **vzor3 - řešení:**

    1. a) $EX = -\frac{3}{10}$, $DX = \frac{141}{10}$

       b) $E\left(5X + X^2\right) = 0$, $D\left(3 - 10X\right) = 141$

       c) $F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \le x < -1 \\ 0.5 & -1 \le x < 0 \\ 0.6 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$

       d) $P(X \ge -1) = 0.8$, $P(X \in \{-2, 0\}) = 0.3$

    2. a) $a = \frac{1}{12}$

       b) $g(y) = \begin{cases} \frac{1}{12}\left(\mathrm{e}^{2y} + 1\right)\mathrm{e}^y & y \in (-\infty, \ln 3) \\ 0 & \text{jinak} \end{cases}$

       c) $P(2 \le X < 5) = \frac{11}{18}$, $P(X < 1) = \frac{1}{9}$

    3. a) $X \sim f_1(x)$, $f_1(x) = \begin{cases} \mathrm{e}^{-x} & x > 0 \\ 0 & \text{jinak} \end{cases}$

       $Y \sim f_2(y)$, $f_2(y) = \begin{cases} \mathrm{e}^{-y} & y > 0 \\ 0 & \text{jinak} \end{cases}$

       b) X a Y jsou stochasticky nezávislé

       c) $P(X = 0) = 0$, $P(X < 1, Y < 1) = \left(1 - \frac{1}{\mathrm{e}}\right)^2$

- **vzor4 - řešení:**

    1. a) $EX = \frac{3}{4}$, $DX = \frac{19}{16}$, $E\left(-3X + X^2\right) = -\frac{1}{2}$, $D\left(5 - 2X\right) = \frac{19}{4}$

       b) $F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{4} & -1 \le x < 1 \\ \frac{3}{4} & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$

       c) $P(X \ge 1) = \frac{3}{4}$, $P\left(X \in \langle -2, 3, \rangle\right) = 1$

    2. a) $a = 0$, $b = -\frac{1}{3}$

       b) $X \sim f(x)$, $f(x) = \begin{cases} \frac{1}{6\sqrt{x}} & 0 < x < 9 \\ 0 & \text{jinak} \end{cases}$

       $Y \sim g(y)$, $g(y) = \begin{cases} \frac{1}{6}\sqrt{\mathrm{e}^y} & y < \ln 9 \\ 0 & \text{jinak} \end{cases}$

       c) $P(0 \le X < 4) = \frac{2}{3}$, $P(X \ge 5) = 1 - \frac{1}{3}\sqrt{5}$

    3. a) $c = \frac{1}{24}$

       b) $X \sim f_1(x)$, $f_1(x) = \begin{cases} \frac{1}{6}(x + 2) & x \in \langle 0, 2 \rangle \\ 0 & \text{jinak} \end{cases}$

       $Y \sim f_2(y)$, $f_2(y) = \begin{cases} \frac{1}{4}y & y \in \langle 1, 3 \rangle \\ 0 & \text{jinak} \end{cases}$

       X a Y jsou stochasticky nezávislé

       c) $P(X = 0, Y > 2) = 0$, $P(X \ge 1, Y < 3) = \frac{7}{12}$

Za správnost výsledků neručím. Pokud najdete jakoukoliv chybu, prosím, ozvěte se mi. Opravím :)