BA002 - jaro 2020, VZOR 1

① a) $\displaystyle\int \frac{1}{\sqrt{1-2x-x^2}}\,dx = \int \frac{1}{\sqrt{2}\cdot\sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}}\,dx = \frac{1}{\sqrt{2}}\cdot\sqrt{2}\arcsin\frac{x+1}{\sqrt{2}} + c = \underline{\underline{\arcsin\frac{x+1}{\sqrt{2}} + c}}$

• úprava na čtverec: $x^2+2x-1 = x^2+2x+1-2 = (x+1)^2-2$

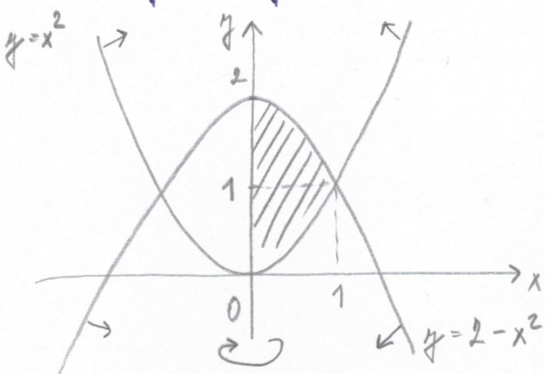
• $\sqrt{1-2x-x^2} = \sqrt{2-(x+1)^2} = \sqrt{2\left(1-\left(\frac{x+1}{\sqrt{2}}\right)^2\right)}$

b) $\displaystyle\int (x^2-1)\sin 2x\,dx = \left| \begin{array}{ll} u(x) = x^2-1 & v'(x) = \sin 2x \\ u'(x) = 2x & v(x) = -\frac{1}{2}\cos 2x \end{array} \right| =$

$\displaystyle = -\frac{1}{2}(x^2-1)\cos 2x + \underbrace{\int x\cos 2x\,dx}_{} = \underline{\underline{-\frac{1}{2}(x^2-1)\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c}}$

$\left| \begin{array}{ll} u(x) = x & v'(x) = \cos 2x \\ u'(x) = 1 & v(x) = \frac{1}{2}\sin 2x \end{array} \right|$

② A: $y \geq x^2,\ y \leq 2-x^2,\ x \geq 0$; rotace kolem osy y



$\displaystyle V_y = \pi\int_c^d f^2(y)\,dy$

$y = x^2 \rightarrow x = \sqrt{y}$

$y = 2-x^2 \rightarrow x = \sqrt{2-y}$

$\displaystyle V_y = \pi\int_0^1 (\sqrt{y})^2\,dy + \pi\int_1^2 (\sqrt{2-y})^2\,dy =$

$\displaystyle = \pi\int_0^1 y\,dy + \pi\int_1^2 (2-y)\,dy = \pi\cdot\frac{1}{2}\left[y^2\right]_0^1 + \pi\left(2\left[y\right]_1^2 - \frac{1}{2}\left[y^2\right]_1^2\right)$

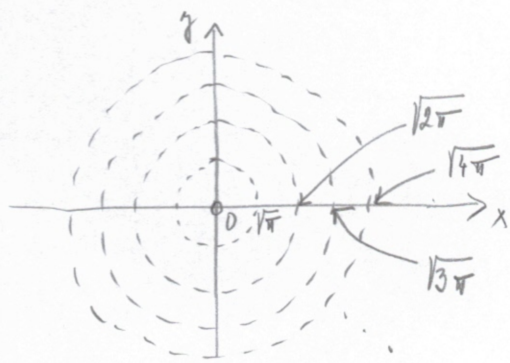
$= \frac{1}{2}\pi + \pi\left(2-\frac{1}{2}\cdot 3\right) = \underline{\underline{\pi}}$

③ $\displaystyle f(x,y) = \frac{1}{\sin(x^2+y^2)}$

$\sin(x^2+y^2) \neq 0$

$x^2+y^2 \neq k\pi,\ k\in\mathbb{Z}$

$D(f) = \{[x,y]\in\mathbb{R}^2 : x^2+y^2 \neq k\pi\}$



④ $\displaystyle f(x,y) = \frac{1}{\sin(x^2+y^2)} = \sin^{-1}(x^2+y^2)$

$f'_x(x,y) = -\sin^{-2}(x^2+y^2)\cdot\cos(x^2+y^2)\cdot 2x$

$f'_y(x,y) = -\sin^{-2}(x^2+y^2)\cdot\cos(x^2+y^2)\cdot 2y$

5. $f(x,y) = \sqrt{\ln x + y^2}$, $A = [1,1]$, $n = 2$, $T_2 = ?$

$f'_x(x,y) = \frac{1}{2}(\ln x + y^2)^{-\frac{1}{2}} \cdot \frac{1}{x} = \frac{1}{2\sqrt{\ln x + y^2}} \cdot \frac{1}{x}$ $\qquad f'_x(A) = \frac{1}{2}$

$f'_y(x,y) = \frac{1}{2}(\ln x + y^2)^{-\frac{1}{2}} \cdot 2y$ $\qquad f'_y(A) = 1$

$f''_{xx}(x,y) = \frac{1}{2} \cdot -\frac{1}{2}(\ln x + y^2)^{-\frac{3}{2}} \cdot \frac{1}{x^2} + \frac{1}{2\sqrt{\ln x + y^2}} \cdot \frac{-1}{x^2}$ $\qquad f''_{xx}(A) = -\frac{1}{4} - \frac{1}{2} = -\frac{3}{4}$

$f''_{xy}(x,y) = \frac{1}{2x} \cdot -\frac{1}{2}(\ln x + y^2)^{-\frac{3}{2}} \cdot 2y$ $\qquad f''_{xy}(A) = -\frac{1}{4} \cdot 2 = -\frac{1}{2}$

$f''_{yy}(x,y) = \frac{1}{2} \cdot -\frac{1}{2}(\ln x + y^2)^{-\frac{3}{2}} \cdot 4y^2 + \frac{1}{2}(\ln x + y^2)^{-\frac{1}{2}} \cdot 2$ $\qquad f''_{yy}(A) = -\frac{1}{4} \cdot 4 + \frac{1}{2} \cdot 2 = 0$

$f(A) = 1$

$T_2(f,A) = 1 + \frac{1}{2}(x-1) + (y-1) + \frac{1}{2}\left[-\frac{3}{4}(x-1)^2 - 2\frac{1}{2}(x-1)(y-1)\right]$

BA002 – jaro 2020, VZOR 2

① a) $\int \frac{\sqrt{x+1}}{\sqrt{x+1}+\sqrt[4]{x+1}}\,dx = \left| \begin{array}{l} x+1 = t^4 \\ dx = 4t^3 dt \end{array} \right| = \int \frac{t^2}{t^2+t}\cdot 4t^3 dt = 4\int \frac{t^4}{t+1}\,dt =$

$t^4 : (t+1) = t^3 - t^2 + t - 1 + \frac{1}{t+1}$

$-(t^4+t^3)$

$-t^3$

$-(-t^3-t^2)$

t^2

$-(t^2+t)$

$-t$

$-(-t-1)$

1

$= 4\left[\frac{1}{4}t^4 - \frac{1}{3}t^3 + \frac{1}{2}t^2 - t + \ln|t+1|\right] + c$

nebo dělení polynomů

b) $\int \frac{\sin^2 x}{\cos x}\,dx = \left| \begin{array}{l} \sin x = t \\ \cos x\,dx = dt \\ dx = \frac{1}{\cos x}dt \end{array} \right| = \int \frac{t^2}{1-t^2}\,dt = -\int \frac{t^2 \pm 1}{t^2-1}\,dt = -\int\left(1+\frac{1}{t^2-1}\right)dt$

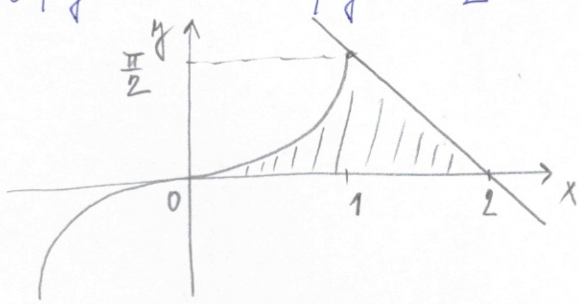
Rozklad na PZ: $\frac{1}{t^2-1} = \frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} \Rightarrow \boxed{1 = A(t+1) + B(t-1)}$

$\Rightarrow A = \frac{1}{2},\ B = -\frac{1}{2}$

$= -\int\left(1 + \frac{\frac{1}{2}}{t-1} - \frac{\frac{1}{2}}{t+1}\right)dt = -t - \frac{1}{2}\ln|t-1| + \frac{1}{2}\ln|t+1| + c$

$= \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| - t + c$, kde $t = \sin x$

② $x = 0,\ y = \arcsin x,\ y = -\frac{\pi}{2}x + \pi$; rotace kolem osy x



$P_x = 2\pi\int_0^1 \arcsin x \sqrt{1+\left(\frac{1}{\sqrt{1-x^2}}\right)^2}\,dx$

$+ 2\pi\int_1^2 \left(-\frac{\pi}{2}x+\pi\right)\sqrt{1+\left(-\frac{\pi}{2}\right)^2}\,dx$

③ $f(x,y) = 3^x \ln(y^2-2)$

$f'_x(x,y) = 3^x \cdot \ln 3 \cdot \ln(y^2-2)$

$f'_y(x,y) = 3^x \cdot \frac{2y}{y^2-2}$

$f''_{yy}(x,y) = 3^x \cdot 2 \cdot \frac{y^2-2-2y\cdot 2y}{(y^2-2)^2} = -2\cdot 3^x \cdot \frac{+3y^2+2}{(y^2-2)^2}$

④ $f(x,y) = \ln(xy) - 4x - 9y$

$$\left.\begin{aligned} f'_x(x,y) &= \frac{\cancel{y}}{x\cancel{y}} - 4 = \frac{1}{x} - 4 = \frac{1-4x}{x} \\ f'_y(x,y) &= \frac{x}{xy} - 9 = \frac{1}{y} - 9 = \frac{1-9y}{y} \end{aligned}\right\} \quad \begin{aligned} \frac{1-4x}{x} &= 0 \\ \frac{1-9y}{y} &= 0 \end{aligned} \quad \Rightarrow x = \frac{1}{4},\ y = \frac{1}{9}$$

$S = [\frac{1}{4}, \frac{1}{9}]$

$f''_{xx}(x,y) = -\frac{1}{x^2}$ $\qquad f''_{xx}(S) = -16$

$f''_{xy}(x,y) = 0$ $\qquad f''_{xy}(S) = 0$

$f''_{yy}(x,y) = -\frac{1}{y^2}$ $\qquad f''_{yy}(S) = -81$

$D(S) = \begin{vmatrix} -16 & 0 \\ 0 & -81 \end{vmatrix} > 0$ ... nastáva extrém

$f''_{xx}(S) = -16 < 0$ ... v S lok. maximum

⑤ $f(x,y) = \ln(xy) - 4x - 9y$ $\qquad f(A) = -13$

$f'_x(x,y) = \frac{1}{x} - 4$ $\qquad f'_x(A) = -3$

$f'_y(x,y) = \frac{1}{y} - 9$ $\qquad f'_y(A) = -8$

$f''_{xx}(x,y) = -\frac{1}{x^2}$ $\qquad f''_{xx}(A) = -1$

$f''_{xy}(x,y) = 0$ $\qquad f''_{xy}(A) = 0$

$f''_{yy}(x,y) = -\frac{1}{y^2}$ $\qquad f''_{yy}(A) = -1$

$$T_2(f,A) = -13 - 3(x-1) - 8(y-1) + \frac{1}{2}\left[-(x-1)^2 - (y-1)^2\right]$$