Příklady na procvičení

1. Vypočtěte dvojný integrál:

(a) $\iint\limits_D \dfrac{1}{(x+y)^2}\,\mathrm{d}x\,\mathrm{d}y, \quad D = \langle 1;2\rangle \times \langle 3;4\rangle,$ $\qquad\left[\ln\dfrac{25}{24}\right]$

(b) $\iint\limits_D \left(\dfrac{1}{x^2}+\dfrac{1}{y^2}\right)\mathrm{d}x\,\mathrm{d}y, \quad D = \langle 3;4\rangle \times \langle 1;2\rangle$ $\qquad\left[\dfrac{7}{12}\right]$

(c) $\iint\limits_D \dfrac{x^2}{1+y^2}\,\mathrm{d}x\,\mathrm{d}y, \quad D = \langle 0;1\rangle \times \langle 0;1\rangle$ $\qquad\left[\dfrac{\pi}{12}\right]$

(d) $\iint\limits_D x^2 y\cos(xy^2)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \left\langle 0;\dfrac{\pi}{2}\right\rangle \times \langle 0;2\rangle$ $\qquad\left[-\dfrac{\pi}{16}\right]$

(e) $\iint\limits_D (x^2+y^2)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \triangle ABC, A[0,0], B[1,0], C[1,1],$ $\qquad\left[\dfrac{1}{3}\right]$

(f) $\iint\limits_D (x-y)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{y\geq 0, x\geq y, x+y\leq 2\},$ $\qquad\left[0\right]$

(g) $\iint\limits_D (x-y)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{y\geq x^2, y\leq x\},$ $\qquad\left[\dfrac{1}{60}\right]$

(h) $\iint\limits_D xy\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{x\geq y, y\geq x^2\},$ $\qquad\left[\dfrac{1}{24}\right]$

(i) $\iint\limits_D (3x-5)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{y\leq 5+x, y\geq -x+7, x\leq 10\},$ $\qquad\left[1296\right]$

(j) $\iint\limits_D (x^2+y)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{y\geq x^2, y\leq \sqrt{x}\},$ $\qquad\left[\dfrac{33}{140}\right]$

(k) $\iint\limits_D \dfrac{x}{3}\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{x\leq 2+\sin y, x\geq 0, y\geq 0, y\leq 2\pi\},$ $\qquad\left[\dfrac{3\pi}{2}\right]$

(l) $\iint\limits_D \dfrac{x^2}{y^2}\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{x\leq 2, \dfrac{1}{x}\leq y\leq x\},$ $\qquad\left[\dfrac{9}{4}\right]$

(m) $\iint\limits_D x\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{x^2+y^2\leq 16, y\geq 0, x\leq 0\},$ $\qquad\left[-\dfrac{64}{3}\right]$

(n) $\iint\limits_D e^x\,\mathrm{d}x\,\mathrm{d}y, \quad D = \{1\leq y\leq 2, 0\leq x\leq \ln y\},$ $\qquad\left[\dfrac{1}{2}\right]$

(o) $\iint\limits_D (x+y)\,\mathrm{d}x\,\mathrm{d}y, \quad D = \Delta ABC, A[0;0], B[4;8], C[4;12],$ $\qquad\left[\dfrac{224}{3}\right]$

(p) $\iint\limits_D (x+y)^2\,\mathrm{d}x\,\mathrm{d}y, \quad D$ je rovnoběžník $ABCD, A[0;1], B[2;-1], C[2;0], D[0;2]$ $\qquad\left[\dfrac{14}{3}\right]$

(q) $\iint\limits_{D}(3x + 8y^2)\,\mathrm{d}x\,\mathrm{d}y,\quad D = \{x^2 + y^2 \geq 1, x^2 + y^2 \leq 4, x \geq 0, y \geq 0\},$

(r) $\iint\limits_{D}(-xy^2)\,\mathrm{d}x\,\mathrm{d}y,\quad D = \{1 \leq x^2 + y^2 \leq 4, x \leq y \leq 0\}.$

2. Vypočtěte obsah plochy omezené podmínkami:

(a) $y \leq \cos x,\ \ 0 \leq x \leq \frac{\pi}{y},\ \ y \geq 0,$

(b) $y \geq \frac{1}{x},\ \ y \geq 4x,\ \ y \leq 8,\ \ y \geq 0,$ $\left[\dfrac{255}{32} - 4\ln 2\right]$

(c) $y \geq x^2,\ \ y \leq 4 - x^2,$ $\left[\dfrac{16\sqrt{2}}{3}\right]$

(d) $y \geq x^2,\ \ y \leq \sqrt{x}.$ $\left[\dfrac{1}{3}\right]$

3. Vypočtěte objem tělesa omezeného podmínkami:

(a) $0 \leq z \leq 9,\ \ y \geq x^2,\ \ x^2 \leq 4 - 3y,$ $[16]$

(b) $x \geq 0,\ \ y \geq 0,\ \ x^2 + y^2 + z^2 \leq 1,\ \ z \geq \sqrt{x^2 + y^2},$ $\left[\dfrac{\pi(2 - \sqrt{2})}{12}\right]$

(c) $x^2 + y^2 \leq 1,\ \ x^2 + y^2 + z^2 \leq 4,$ $\left[4\pi\left(\dfrac{8}{3} - \sqrt{3}\right)\right]$

(d) $x^2 - y - 2 \leq 0,\ \ 2 \geq x^2 + y^2,$

(e) $x^2 + y^2 + y^2 \geq 1,\ \ x^2 + y^2 + z^2 \leq 4, 3z^2 \geq x^2 + y^2,\ \ z \geq 0,$ $\left[\dfrac{7}{3}\pi\right]$

(f) $x \geq 0,\ \ y \geq 0,\ \ z \geq 0,\ \ x + y + z \leq 1,$

(g) $z \leq x^2 + y^2,\ \ x + y \leq 1,\ \ x \geq 0,\ \ y \geq 0,\ \ z \geq 0,$

(h) $z \leq 4 - x^2 - y^2,\ \ 2x \geq 2 + x^2 + y^2,$

(i) $x^2 + y^2 + z^2 \leq 4,\ \ x^2 + y^2 \leq 3z.$

4. Vypočtěte trojný integrál:

(a) $\iiint\limits_{D} xyz\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $\left[\dfrac{4}{3}\right]$

(b) $\iiint\limits_{D} z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $[\pi]$

(c) $\iiint\limits_{D} z^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $\left[\dfrac{16}{15}\pi\right]$

(d) $\iiint\limits_{D} x\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $[\pi]$

(e) $\iiint\limits_{D} xy\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $\left[\dfrac{32}{15}\right]$

(f) $\iiint\limits_{D} y\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $[\pi]$

(g) $\iiint\limits_{D} x^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $\left[\dfrac{16}{15}\pi\right]$

(h) $\iiint\limits_{D} y^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\quad D = \{x^2 + y^2 + z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\},$ $\left[\dfrac{16}{15}\pi\right]$

(i) $\iiint\limits_D (x^2 + y^2)\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{15}\pi\right]$

(j) $\iiint\limits_D xz\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{15}\right]$

(k) $\iiint\limits_D yz\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{15}\right]$

(l) $\iiint\limits_D xz^2\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{2\pi}{3}\right]$

(m) $\iiint\limits_D yz^2\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{2\pi}{3}\right]$

(n) $\iiint\limits_D x^2 z\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{2\pi}{3}\right]$

(o) $\iiint\limits_D (x^2 + y^2 + z^2)\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{16}{5}\pi\right]$

(p) $\iiint\limits_D (x^2 + z^2)\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{15}\pi\right]$

(q) $\iiint\limits_D (y^2 + z^2)\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{15}\pi\right]$

(r) $\iiint\limits_D (x^2 + y^2)z\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{4}{3}\pi\right]$

(s) $\iiint\limits_D (x^2 + y^2)z^2\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{128}{105}\pi = \dfrac{2^7}{105}\pi\right]$

(t) $\iiint\limits_D (x^2 + y^2)z^3\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{35}\pi\right]$

(u) $\iiint\limits_D (x + y)z\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{64}{15}\right]$

(v) $\iiint\limits_D (x + y)z^2\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{4}{3}\pi\right]$

(w) $\iiint\limits_D (x + y)z^3\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{512}{105} = \dfrac{2^9}{105}\right]$

(x) $\iiint\limits_D (x + y)^2\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$, $\qquad \left[\dfrac{32}{5}(\pi + 2)\right]$

(a) $\iiint\limits_D x\,dx\,dy\,dz$, $\quad D = \{x + 2y + z \le 1, x > 0,\ y \ge 0,\ z \ge 0\}$,

(b) $\iiint\limits_D x\,dx\,dy\,dz$, $\quad D = \{x \ge 0,\ y \ge 0,\ x + y \le 1,\ 0 \le z \le x^2 + y^2 + 1\}$,

(c) $\iiint\limits_D (x + y + z)\,dx\,dy\,dz$, $\quad D = \{0 \le x \le 3,\ 0 \le y \le 2,\ 0 \le z \le 1\}$.

(a) $\iiint\limits_D x\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge y,\ z \ge 0\}$,

(b) $\iiint\limits_D y\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge y,\ z \ge 0\}$,

(c) $\iiint\limits_D z\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge y,\ z \ge 0\}$,

(d) $\iiint\limits_D xy\,dx\,dy\,dz$, $\quad D = \{x^2 + y^2 + z^2 \le 4,\ x \ge y,\ z \ge 0\}$,

(e) $\iiint\limits_{D} yz \, dx \, dy \, dz, \quad D = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$

(f) $\iiint\limits_{D} xz \, dx \, dy \, dz, \quad D = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$

(g) $\iiint\limits_{D} xyz \, dx \, dy \, dz, \quad D = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$

(a) $\iiint\limits_{W} \dfrac{z}{\sqrt{x^2 + y^2}} \, dx \, dy \, dz, \quad W = \{1 \leq x^2 + y^2 + z^2 \leq 9, \ x \geq y, \ z \geq 0\},$ $\qquad \left[\dfrac{26}{3}\pi\right]$

(b) $\iiint\limits_{K} \dfrac{z}{\sqrt{x^2 + y^2}} \, dx \, dy \, dz, \quad K = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$

(c) $\iiint\limits_{W} \dfrac{x}{\sqrt{x^2 + y^2}} \, dx \, dy \, dz, \quad W = \{1 \leq x^2 + y^2 + z^2 \leq 9, \ x \geq y, \ z \geq 0\},$ $\qquad [\ ]$

(d) $\iiint\limits_{K} \dfrac{x}{\sqrt{x^2 + y^2}} \, dx \, dy \, dz, \quad K = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$

(e) $\iiint\limits_{W} \dfrac{y}{\sqrt{x^2 + y^2}} \, dx \, dy \, dz, \quad W = \{1 \leq x^2 + y^2 + z^2 \leq 9, \ x \geq y, \ z \geq 0\},$ $\qquad [\ ]$

(f) $\iiint\limits_{K} \dfrac{y}{\sqrt{x^2 + y^2}} \, dx \, dy \, dz, \quad K = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$

(g) $\iiint\limits_{W} z\sqrt{x^2 + y^2} \, dx \, dy \, dz, \quad W = \{1 \leq x^2 + y^2 + z^2 \leq 9, \ x \geq y, \ z \geq 0\},$ $\qquad [\ ]$

(h) $\iiint\limits_{K} z\sqrt{x^2 + y^2} \, dx \, dy \, dz, \quad K = \{x^2 + y^2 + z^2 \leq 4, \ x \geq y, \ z \geq 0\},$