Příklady ke 2. zápočtové písemce

Limita funkce

Určete limitu funkce (pokud linita neexistuje, zdůvodněte):

1) $\lim_{x \to 0} \dfrac{x}{\cos x - 1} =$

2) $\lim_{x \to 0} \dfrac{x}{\cos^2 x - 1} =$

3) $\lim_{x \to 0} \dfrac{x}{(\cos x - 1)^2} =$

4) $\lim_{x \to 0} \dfrac{1}{\cos x - 1} =$

5) $\lim_{x \to 0} \dfrac{1}{\sin x} =$

6) $\lim_{x \to \infty} \dfrac{1}{\sin x} =$

7) $\lim_{x \to \infty} \dfrac{1}{\sin^2 x} =$

Taylorův a MacLaurinův polynom

Průběh funkce

Řešte průběh funkce:

1) $f(x) = \dfrac{x^2 - 6x}{x + 2},$

2) $f(x) = \dfrac{2x^2 - 5x}{x + 2},$

3) $f(x) = \dfrac{x^2 - x - 2}{x + 2},$

8) $f(x) = \dfrac{1}{\sin x}.$

Soustavy lineárních rovnic

Řešte soustavu lineárních rovnic:

1) $2x_1 + x_2 + x_4 = 3,$
$-4x_1 + 4x_2 - x_3 - x_4 = 7,$
$-2x_1 + x_2 + 2x_3 + x_4 = -5.$
$2x_1 - 2x_2 - 2x_3 - x_4 = 3,$

Determinant matice

Určete determinant matice a zda je matice regulární nebo singulární:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \\ 1 & -2 & 3 \end{pmatrix},$$

$$D=\begin{pmatrix}0&-4&3\\-5&1&-1\\2&-2&1\end{pmatrix},\qquad F=\begin{pmatrix}1&1&1\\2&1&1\\2&2&2\end{pmatrix},\qquad G=\begin{pmatrix}2&1&1\\2&2&1\\2&2&2\end{pmatrix},$$

$$H=\begin{pmatrix}2&2&2\\1&2&2\\1&1&2\end{pmatrix},\qquad J=\begin{pmatrix}-2&-2&-2\\1&-2&-2\\1&1&-2\end{pmatrix},\qquad K=\begin{pmatrix}-2&2&-2\\-2&-2&1\\-2&1&1\end{pmatrix},$$

$$L=\begin{pmatrix}-2&-2&-2\\-2&-1&1\\-2&1&1\end{pmatrix},\qquad M=\begin{pmatrix}1&-4&2\\5&-3&0\\5&2&-3\end{pmatrix},\qquad N=\begin{pmatrix}6&0&1\\-3&2&3\\4&-2&-3\end{pmatrix},$$

$$P=\begin{pmatrix}5&2&1\\3&1&6\\-4&-2&10\end{pmatrix},\qquad Q=\begin{pmatrix}2&2&1&2\\3&1&-4&1\\2&-2&-5&0\\1&-1&2&3\end{pmatrix},\qquad R=\begin{pmatrix}2&4&1&2\\3&1&-4&1\\2&-2&-5&0\\1&-1&2&3\end{pmatrix},$$

$$S=\begin{pmatrix}2&2&\frac{1}{2}&1\\\frac{3}{2}&-\frac{1}{2}&2&\frac{1}{2}\\1&-1&\frac{5}{2}&0\\\frac{1}{2}&-\frac{1}{2}&1&\frac{3}{2}\end{pmatrix},\qquad T=\begin{pmatrix}2&2&\frac{1}{2}&1\\\frac{3}{2}&-\frac{1}{2}&2&\frac{1}{2}\\2&-2&4&-6\\-\frac{1}{2}&\frac{1}{2}&-1&\frac{3}{2}\end{pmatrix}.$$

NUMERICKÁ MATEMATIKA

Numerické derivování

Metoda sečen a metoda tečen

LU-rozklad

Výběr hlavního prvku

Řešení

Limita funkce

1) neexistuje,

2) neexistuje,

3) neexistuje,

4) $-\infty$,

5) neexistuje,

6) neexistuje,

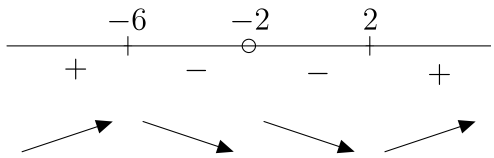
7) ∞,

Taylorův a MacLaurinův polynom

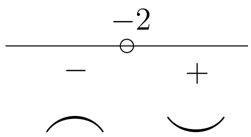
Průběh funkce

1) $f(x) = \dfrac{x^2 - 6x}{x + 2} = \dfrac{x(x-6)}{x+2}$:

- $\mathrm{D}(f) = \mathbb{R} - \{-2\}$,
- průsečíky s osami: [0, 0], [6, 0],
- není sudá, ani lichá, ani periodická,
- $f' = \dfrac{x^2 + 4x - 12}{(x+2)^2} = \dfrac{(x-2)(x+6)}{(x+2)^2}$ $\Rightarrow$ NB: -6, -2, 2, $\Rightarrow$



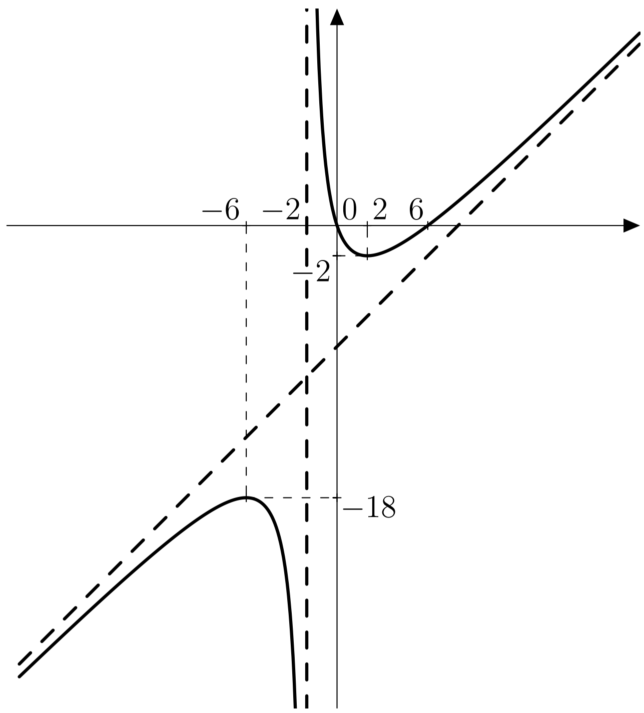
- max:[-6, -18], min:[2, -2],
- $f'' = \dfrac{32}{(x+2)^3}$ $\Rightarrow$ NB: -2, $\Rightarrow$



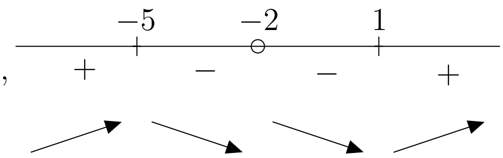
$\Rightarrow$ nemá inflexní body,

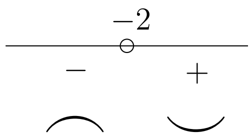
- asymptoty bez směrnice: $x = -2$ $\quad \left[\lim\limits_{x\to -2^-} f(x) = -\infty, \quad \lim\limits_{x\to -2^+} f(x) = \infty\right]$,

  asymptoty se směrnicí: $y = x - 8$ pro $x \to \pm\infty$,
- graf:

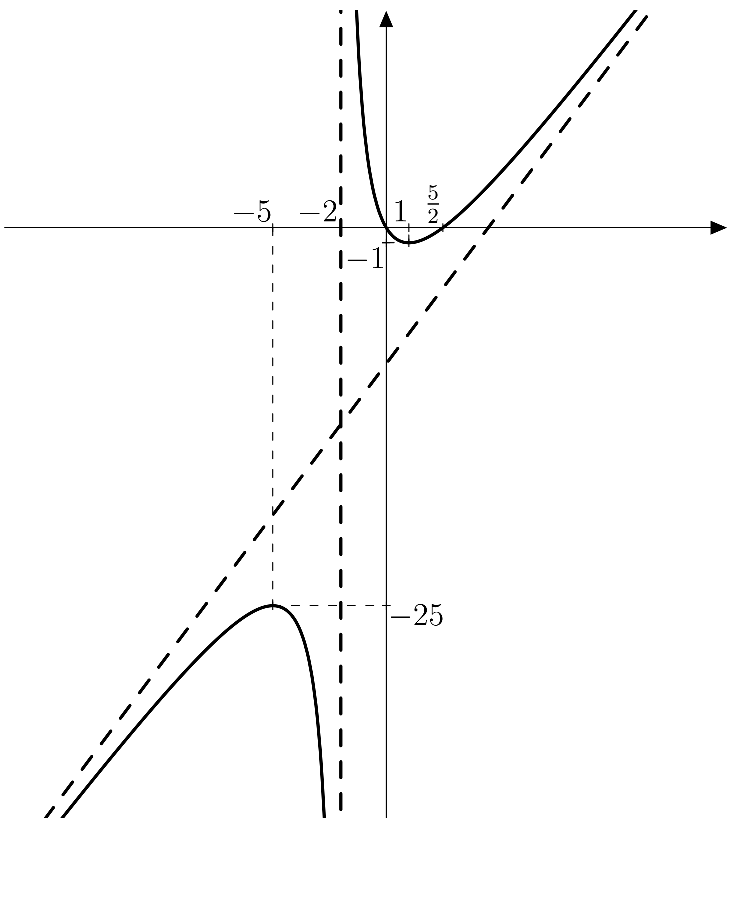


2) $f(x) = \dfrac{2x^2 - 5x}{x+2} = \dfrac{x(2x-5)}{x+2}$:

- $\mathrm{D}(f) = \mathbb{R} - \{-2\}$,
- průsečíky s osami: $[0,\, 0]$, $[\frac{5}{2},\, 0]$,
- není sudá, ani lichá, ani periodická,
- $f' = \dfrac{2x^2 + 8x - 10}{(x+2)^2} = \dfrac{2(x-1)(x+5)}{(x+2)^2} \;\Rightarrow$ NB: -5, -2, 1, 

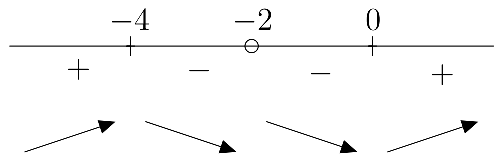
$\Rightarrow$
- max:[-5, -25], min:[1, -1],
- $f'' = \dfrac{36}{(x+2)^3} \;\Rightarrow\;$ NB: -2, $\Rightarrow$ 

$\Rightarrow$ nemá inflexní body,
- asymptoty bez směrnice: $x = -2$ $\left[\lim\limits_{x\to -2^-} f(x) = -\infty, \quad \lim\limits_{x\to -2^+} f(x) = \infty\right]$,
  asymptoty se směrnicí: $y = 2x - 9$ pro $x \to \pm\infty$,
- graf:

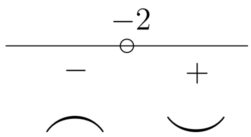


3) $f(x) = \dfrac{x^2 - x - 2}{x+2} = \dfrac{(x+1)(x-2)}{x+2}$:

- $\mathrm{D}(f) = \mathbb{R} - \{-2\}$,
- průsečíky s osami: [-1, 0], [2, 0], [0, -1],
- není sudá, ani lichá, ani periodická,
- $f' = \dfrac{x^2 + 4x}{(x+2)^2} = \dfrac{x(x+4)}{(x+2)^2}$ $\Rightarrow$ NB: -4, -2, 0, $\Rightarrow$



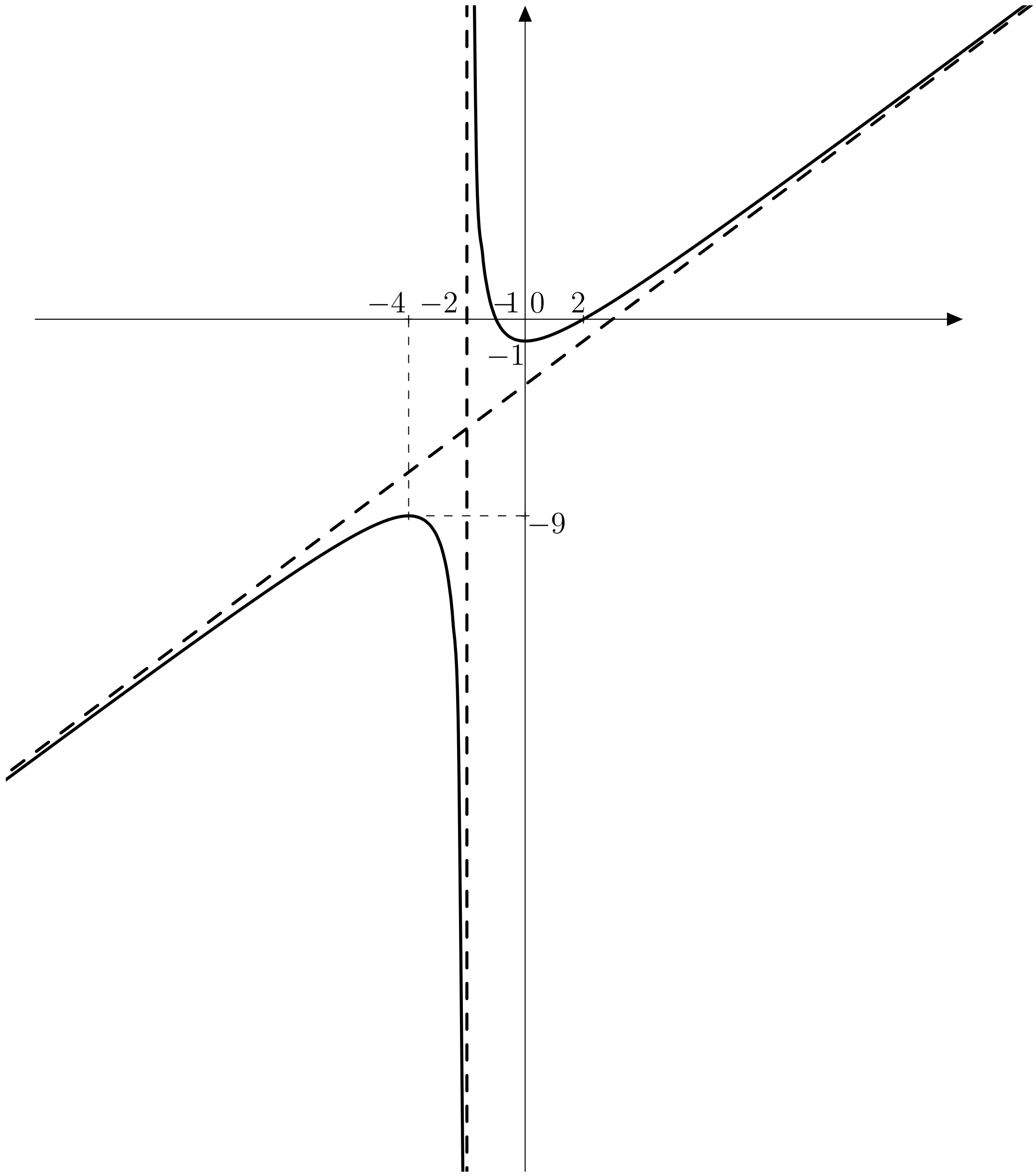
- max:[-4, -9], min:[0, -1],
- $f'' = \dfrac{8}{(x+2)^3}$ $\Rightarrow$ NB: -2, $\Rightarrow$



$\Rightarrow$ nemá inflexní body,

- asymptoty bez směrnice: $x = -2$ $\left[\lim_{x\to -2^-} f(x) = -\infty, \quad \lim_{x\to -2^+} f(x) = \infty\right]$,

  asymptoty se směrnicí: $y = x - 3$ pro $x \to \pm\infty$,
- graf:



4) $f(x) = \dfrac{2x^2 + 3x}{x + 2}$.

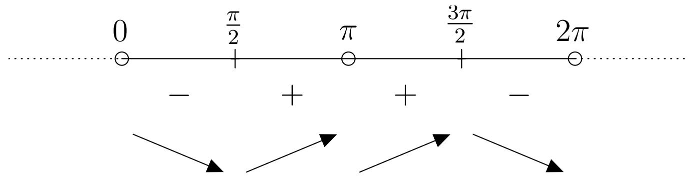
5) $f(x) = \dfrac{x^2 + 2x + 1}{x + 2}$.

6) $f(x) = \dfrac{x^2 - 6x - 7}{x + 2}$.

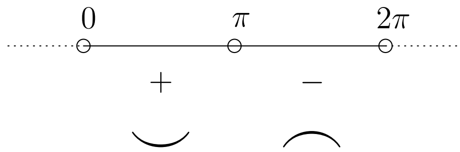
7) $f(x) = \dfrac{3x^2 + 2x - 5}{x + 2}$.

8) $f(x) = \dfrac{1}{\sin x}$:

- $\mathrm{D}(f) = \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}$,
- průsečíky s osami: nemá,
- lichá a periodická s periodou $p = 2\pi$,
- $f' = -\dfrac{\cos x}{\sin^2 x} \;\Rightarrow\;$ NB: $k\frac{\pi}{2}$, $\;\Rightarrow$

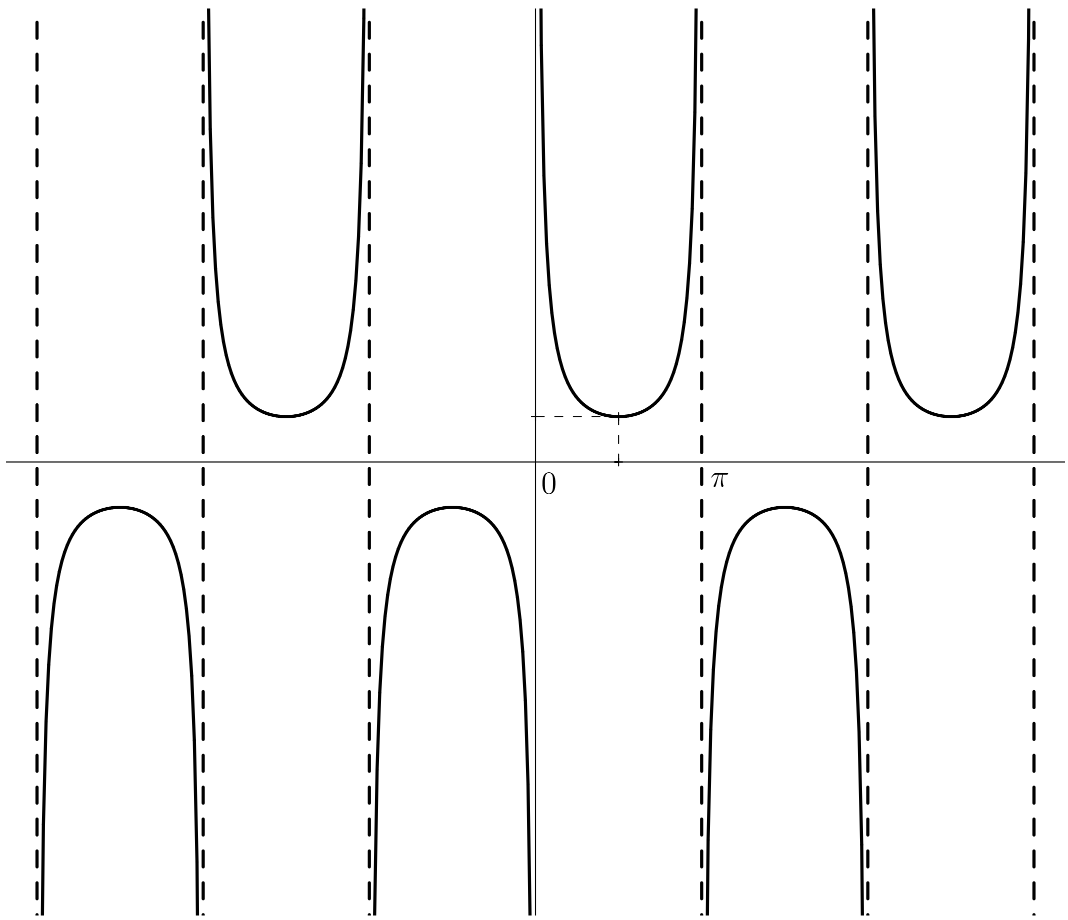


- max: $[\frac{3\pi}{2} + 2k\pi$, -1], min: $[\frac{\pi}{2} + 2k\pi$, 1],
- $f'' = \dfrac{1 + \cos^2 x}{\sin^3 x} \;\Rightarrow\;$ NB: $k\pi$, $\Rightarrow$



$\Rightarrow$ nemá inflexní body,

- asymptoty bez směrnice: $x = k\pi \quad \left[\lim\limits_{x \to k\pi^-} f(x) = -\infty, \quad \lim\limits_{x \to k\pi^+} f(x) = \infty\right]$,
  asymptoty se směrnicí: nemá,
- graf:



Soustavy lineárních rovnic

1) $(x_1, x_2, x_3, x_4) = (1, 2, -2, -1)$,

Determinant matice

$|A| = -1, \quad |B| = 0, \quad |C| = 32,$

$|D| = 12, \quad |F| = 1, \quad |G| = 2,$

$|H| = 2, \quad |J| = -18, \quad |K| = 18,$

$|L| = 12, \quad |M| = -1, \quad |N| = -2,$

$|P| = 0, \quad |Q| = -24, \quad |R| = 0,$

$|S| = -\frac{7}{2}, \quad |T| = 0.$