Příklady k 1. zápočtové písemce

Definiční obor funkce

Určete definiční obor funkce:

1) $f(x) = \sqrt{\dfrac{x^2+2x-15}{x^2+4x-12}}$.

2) $f(x) = \log_2\left(\dfrac{x^2+4x-12}{x^2+2x-15}\right)$.

3) $f(x) = \sqrt{\dfrac{x^2+x-12}{x^3+3x^2-10x}}$.

4) $f(x) = \log_2\left(\dfrac{x^3+3x^2-10x}{x^2+x-12}\right)$.

5) $f(x) = \sqrt{\dfrac{-x^3+5x^2-8x+4}{x^2+x-12}}$.

6) $f(x) = \log_2\left(\dfrac{x^2+x-12}{-x^3+5x^2-8x+4}\right)$.

7) $f(x) = \sqrt{\dfrac{x^2-3x+1}{3x^2+7x-6}}$.

8) $f(x) = \log_2\left(\dfrac{3x^2+7x-6}{x^2-3x+1}\right)$.

9) $f(x) = \sqrt{\dfrac{x^2-3x-10}{x^2+5x+6}}$.

10) $f(x) = \log_2\left(\dfrac{x^2+5x+6}{x^2-3x-10}\right)$.

11) $f(x) = \sqrt{\dfrac{x^3-4x^2-3x+18}{x^2-5x+6}}$.

12) $f(x) = \log_2\left(\dfrac{x^2-5x+6}{x^3-4x^2-3x+18}\right)$.

Funkce inverzní

K dané funkci $f(x)$ určete inverzní funkci, $\mathrm{D}(f)$, $\mathrm{D}(f^{-1})$, $\mathrm{H}(f)$ a $\mathrm{H}(f^{-1})$:

Znaménko racionálně lomené funkce

Určete znaménko a $\mathrm{D}(f)$ funkce:

1) $f(x) = \dfrac{x^2-3x+1}{2x^2+3x-2}$.

2) $f(x) = \dfrac{x^2+3x-18}{x^2+3x-10}$.

3) $f(x) = \dfrac{x^3+4x^2-5x}{x^2+x-6}$.

4) $f(x) = \dfrac{x^3 + 3x^2 - 9x + 5}{6 + x - x^2}$.

5) $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 5x + 6}$.

6) $f(x) = \dfrac{x^3 - 3x + 2}{x^2 - 3x + 2}$.

7) $f(x) = \dfrac{x^3 - x^2 + 2x - 2}{2x^2 - 5x - 12}$.

8) $f(x) = \dfrac{x^3 - x - 6}{3x^2 - 5x - 2}$.

9) $f(x) = \dfrac{6x^2 - x - 1}{x^3 + 2x^2 + 4x + 3}$.

10) $f(x) = \dfrac{x^2 - 2x - 8}{x^4 - 2x^2 + 1}$.

11) $f(x) = \dfrac{x^4 - 8x^2 + 16}{-x^2 + 4x + 5}$.

12) $f(x) = \dfrac{2x^2 + 3x - 1}{x^2 - x + 2}$.

13) $f(x) = \dfrac{x^2 - 2x - 2}{2x - 3 - x^2}$.

14) $f(x) = \dfrac{1 - 2x - x^2}{x^5 - x^4 - 6x^3}$.

15) $f(x) = \dfrac{-x^3 + 2x^2 - 4x + 3}{2x^3 + 4x^2 + 8x + 16}$.

16) $f(x) = \dfrac{-x^3 + 2x^2 + 5x - 6}{x^2 - x - 2}$.

17) $f(x) = \dfrac{x^2 + 2x - 8}{x^3 + 3x^2 - 6x - 8}$.

18) $f(x) = \dfrac{x^3 + x^2 - 2x}{x^3 + 2x^2 - x - 2}$.

19) $f(x) = \dfrac{-x^3 + x^2 + 17x + 15}{x^2 - 4x - 5}$.

20) $f(x) = \dfrac{x^4 + 3x^2 + 2}{x^3 + 8}$.

21) $f(x) = \dfrac{x^2 - 3x + 2}{x^4 - x^2 - 2}$.

22) $f(x) = \dfrac{x^4 - 2}{x^4 - 4x^2 + 4}$.

23) $f(x) = \dfrac{x^5 + 1}{x^4 - 1}$.

24) $f(x) = \dfrac{1-x^3}{x^4+1}$.

25) $f(x) = \dfrac{-x^4+x^3+6x^2-15x+9}{x^3-x^2-3x+6}$.

26) $f(x) = \dfrac{4x^3-\frac{4}{3}x^2-9x+3}{x^2+5x-6}$.

27) $f(x) = \dfrac{x^3-8}{x^2+\frac{5}{6}x-1}$.

28) $f(x) = \dfrac{5-3x-x^2}{2x^3-23x^2-15x+36}$.

29) $f(x) = \dfrac{x^3-52x+96}{x^3-3x^2-3x}$.

30) $f(x) = \dfrac{-x^3+4x^2-6x+3}{3x^3+2}$.

31) $f(x) = \dfrac{(2x^2+x-1)(x+1)}{x^2+x+4}$.

32) $f(x) = \dfrac{(x^2-1)(x+1)}{x^2+x-6}$.

33) $f(x) = \dfrac{(x^4-3x^3)(x^2+2x+8)}{x+3}$.

34) $f(x) = \dfrac{3x^3-5x^2+2x}{x^2+4x-5}$.

35) $f(x) = \dfrac{(x^2-3x+2)(x+3)}{x^2+x+1}$.

36) $f(x) = \dfrac{(x-2)(x^2+1)}{x^2+5x+6}$.

37) $f(x) = \dfrac{x-\frac{1}{x}}{x+\frac{3x+1}{x-1}}$.

Limita funkce

Určete limitu funkce:

1) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x^3-x^2} =$

2) $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-2x-3} =$

3) $\lim\limits_{x\to \infty} \dfrac{x^2+5x+4}{x^2-2x+3} =$

4) $\lim\limits_{x\to 0} \dfrac{\sin x}{\sin(2x)} =$

5) $\lim\limits_{x\to -2} \dfrac{2x^2+5x+2}{x+2} =$

6) $\lim\limits_{x\to\infty} \dfrac{2x^2+5x-1}{x^3-x^2+1} =$

7) $\lim\limits_{x\to-3} \dfrac{2x^2+3x-9}{x+3} =$

NE na písemku:

*) $\lim\limits_{x\to-\infty} \left[x\left(\sqrt{x^2+1}+x\right)\right] =$

Derivace funkce

Zderivujte funkci a určete její definiční obor:

1) $f(x) = (x-2)\sqrt{1-\mathrm{e}^x}$

2) $f(x) = \dfrac{\cos x}{3(1+\sin x)}$

3) $f(x) = \mathrm{e}^{-2x} \cdot \cos(3x)$

4) $f(x) = \dfrac{-\cos x}{2\sin^2 x}$

5) $f(x) = \log_2 \sqrt{1-x^2} + \frac{1}{x}$

6) $f(x) = \ln \dfrac{x+1}{x-1}$

7) $f(x) = \dfrac{x^3 \cdot \sin x}{\cos x - 1}$

8) $f(x) = \log \sqrt{1-x^2} =$

9) $f(x) = \sqrt{1+\left(\dfrac{x}{\sqrt{1-x^2}}\right)^2} =$

10) $f(x) = \dfrac{\sin x + 1}{\cos x} =$

Tečna ke grafu funkce

Napište rovnici tečny a normály ke grafu funkce $f(x)$ v bodě T:

1) $f(x) = x^2+4x+8, \quad T[0, ?].$

2) $f(x) = \operatorname{arctg} x, \quad T[?, \frac{\pi}{4}].$

3) $f(x) = 2\sqrt{2}\sin x, \quad T[\frac{\pi}{6}, ?].$

4) $f(x) = \mathrm{e}^{2(x-1)}, \quad T[?, 1].$

5) $f(x) = x^2, \quad T[3, ?].$

6) $f(x) = x^2+2x, \quad T[4, ?].$

7) $f(x) = \dfrac{-2x}{(x^2+1)^2}, \quad T[1, ?].$

8) $f(x) = \mathrm{e}^{2x}, \quad T[0, ?]$.

9) $f(x) = \ln x, \quad T[\mathrm{e}, ?]$.

NUMERICKÁ MATEMATIKA

Metoda bisekce (půlení intervalů)

S chybou menší než ε najděte přibližné řešení rovnice:

1)$= 0$.

Metoda regula-falsi

Interpolační polynom

Nalezněte interpolační polynom procházející body

1) [-2, 3], [-1, -2], [1, 2], [2, -4].

2) [-2, 3], [-1, 2], [1, 2], [2, -3].

3) [-2, 0], [-1, -5], [1, 2], [2, -4].

4) [0, 0], [-1, -2], [1, 2], [2, -2].

5) [1, -2], [-2, -1], [2, 1], [-4, 2].

6) [-5, -2], [-2, 0], [2, 10], [5, 2].

7) [4, -10], [-2, 0]. [-4, -2], [0, 2], [2, 0].

8) [-3, 2], [1, 2], [-1, 2], [2, 3].

9) [3, -4], [1, 2], [-1, 2], [-3, 2].

10) [-1, 2], [-5, 2], [0, 2], [1, -4].

11) [1, -4], [0, 2], [-5, 2], [-1, 8].

12) [-1, 2], [-5, 2], [-5, 4], [0, 2].

13) [-1, 2], [$\frac{1}{2}$, $\frac{1}{2}$], [0, 2], [-2, -4].

14) [-1, -2], [0, 2], [1, 6], [2, 4].

15) [-1, -2], [0, 2], [1, 6], [2, 4].

16[-1, 2], [-$\frac{3}{2}$, $\frac{1}{2}$], [0, 2], [-2, -4].

Řešení

Definiční obor funkce

1) $f(x) = \sqrt{\dfrac{(x-3)(x+5)}{(x-2)(x+6)}}, D(f) = .$

2) $f(x) = \log_2 \left(\dfrac{(x-2)(x+6)}{(x-3)(x+5)} \right).$

3) $f(x) = \sqrt{\dfrac{(x-3)(x+4)}{x(x-2)(x+5)}}, D(f) = .$

4) $f(x) = \log_2 \left(\dfrac{x(x-2)(x+5)}{(x-3)(x+4)} \right).$

5) $f(x) = \sqrt{\dfrac{(x-2)^2(x-1)}{(3-x)(x+4)}}, D(f) = .$

6) $f(x) = \log_2 \left(\dfrac{(3-x)(x+4)}{(x-2)^2(x-1)} \right).$

7) $f(x) = \sqrt{\dfrac{(x - \frac{3+\sqrt{5}}{2})(x - \frac{3-\sqrt{5}}{2})}{(3x-2)(x+3)}}, D(f) = .$

8) $f(x) = \log_2 \left(\dfrac{(3x-2)(x+3)}{(x - \frac{3+\sqrt{5}}{2})(x - \frac{3-\sqrt{5}}{2})} \right).$

9) $f(x) = \sqrt{\dfrac{(x+2)(x-5)}{(x+2)(x+3)}} = \sqrt{\dfrac{x-5}{x+3}}, D(f) = .$

10) $f(x) = \log_2 \left(\dfrac{(x+2)(x+3)}{(x+2)(x-5)} \right) = \log_2 \left(\dfrac{x+3}{x-5} \right).$

11) $f(x) = \sqrt{\dfrac{(x-3)^2(x+2)}{(x-3)(x-2)}} = \sqrt{\dfrac{(x-3)(x+2)}{x-2}}, D(f) = .$

12) $f(x) = \log_2 \left(\dfrac{(x-3)(x-2)}{(x-3)^2(x+2)} \right) = \log_2 \left(\dfrac{x-2}{(x-3)(x+2)} \right).$

13) $f(x) = \sqrt{\dfrac{x - \frac{1}{x}}{x + \frac{3x+1}{x-1}}} = \sqrt{\dfrac{\frac{x^2-1}{x}}{\frac{x^2-1+3x+1}{x-1}}} = \sqrt{\dfrac{x^2-1}{x} \cdot \dfrac{x-1}{x^2+3x}} = \sqrt{\dfrac{(x-1)^2(x+1)}{x^2(x+3)}},$

$D(f) = (-\infty; -3) \cup \langle -1; 0) \cup (0; \infty)$ [viz příklad 37/ Rozklad racionálně lomené funkce].

14) $f(x) = \log_5 \dfrac{x - \frac{1}{x}}{x + \frac{3x+1}{x-1}} = \cdots = \log_5 \dfrac{(x-1)^2(x+1)}{x^2(x+3)},$

$D(f) = (-\infty; -3) \cup (-1; 0) \cup (0; 1) \cup (1; \infty)$ [viz příklad 37/ Rozklad racionálně lomené funkce].

Znaménko racionálně lomené funkce

1) $f(x) = \dfrac{(x - \frac{3+\sqrt{5}}{2})(x - \frac{3-\sqrt{5}}{2})}{(2x-1)(x+2)}.$

2) $f(x) = \dfrac{(x-2)(x+5)}{(x-3)(x+6)}.$

3) $f(x) = \dfrac{x(x-1)(x+5)}{(x-2)(x+3)}.$

4) $f(x) = \dfrac{(x-1)^2(x+5)}{(3-x)(x+2)}.$

5) $f(x) = \dfrac{(x-3)(x+5)}{(x-3)(x-2)} = \dfrac{x+5}{x-2}.$

6) $f(x) = \dfrac{(x-1)^2(x+2)}{(x-1)(x-2)} = \dfrac{(x-1)(x+2)}{x-2}.$

7) $f(x) = \dfrac{(x-1)(x^2+2)}{(x-4)(2x+3)}.$

8) $f(x) = \dfrac{(x-2)(x^2+2x+3)}{(x-2)(3x+1)} = \dfrac{x^2+2x+3}{3x+1}.$

9) $f(x) = \dfrac{(2x-1)(3x+1)}{(x+1)(x^2+x+3)}.$

10) $f(x) = \dfrac{(x-4)(x+2)}{(x+1)^2(x-1)^2}.$

11) $f(x) = \dfrac{(x+2)^2(x-2)^2}{(5-x)(x+1)}.$

12) $f(x) = \dfrac{2(x+\frac{3+\sqrt{17}}{4})(x+\frac{3-\sqrt{17}}{4})}{x^2-x+2}.$

13) $f(x) = \dfrac{(x-1-\sqrt{3})(x-1+\sqrt{3})}{2x-3-x^2}.$

14) $f(x) = \dfrac{-(x+1+\sqrt{2})(x+1-\sqrt{2})}{x^3(x-3)(x+2)}.$

15) $f(x) = \dfrac{(x^2-x+3)(1-x)}{2(x^2+4)(x+2)}.$

16) $f(x) = \dfrac{(x+2)(1-x)(x-3)}{(x+1)(x-2)}.$

17) $f(x) = \dfrac{(x-2)(x+4)}{(x+1)(x-2)(x+4)} = \dfrac{1}{x+1}.$

18) $f(x) = \dfrac{x(x-1)(x+2)}{(x+1)(x-1)(x+2)}.$

19) $f(x) = \dfrac{(x+3)(x+1)(5-x)}{(x+1)(x-5)} = -x-3.$

20) $f(x) = \dfrac{(x^2+2)(x^2+1)}{(x+2)(x^2-2x+4)}.$

21) $f(x) = \dfrac{(x-2)(x-1)}{(x+\sqrt{2})(x-\sqrt{2})(x^2+1)}.$

22) $f(x) = \dfrac{(x-\sqrt[4]{2})(x+\sqrt[4]{2})(x^2+\sqrt{2})}{(x-\sqrt{2})^2(x+\sqrt{2})^2}.$

23) $f(x) = \dfrac{(x+1)(x^4-x^3+x^2-x+1)}{(x-1)(x+1)(x^2+1)} = \dfrac{x^4-x^3+x^2-x+1}{(x-1)(x^2+1)}$
[čitatel lze ještě rozložit, ale zbývající 4 kořeny jsou všechny komplexní; viz graf funkce x^5+1].

24) $f(x) = \dfrac{(1-x^3)(1+x+x^2)}{x^4+1}$
[jmenovatel lze ještě rozložit, ale zbývající 4 kořeny jsou všechny komplexní; viz graf x^4+1];
$\big($porovnání koeficientů: $x^4+1 = (x^2+ax+b)(x^2+cx+d) = \ldots = (x^2+\sqrt{2}x+1)(x^2-\sqrt{2}+1)\big)$.

25) $f(x) = \dfrac{(x^2-3x+3)(1-x)(x+3)}{(x+2)(x^2-3x+3)} = \dfrac{(1-x)(x+3)}{x+2}.$

26) $f(x) = \dfrac{(3x-1)(2x-3)(2x+3)}{3(x-1)(x+6)}.$

27) $f(x) = \dfrac{6(x-2)(x^2+2x+4)}{(3x-2)(2x+3)}.$

28) $f(x) = \dfrac{(x+\frac{3+\sqrt{29}}{2})(x+\frac{3-\sqrt{29}}{2})}{(x-12)(2x+3)(x-1)}.$

29) $f(x) = \dfrac{(x-6)(x-2)(x+8)}{x(x-\frac{3-\sqrt{21}}{2})(x-\frac{3+\sqrt{21}}{2})}.$

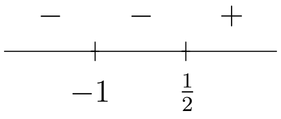
30) $f(x) = \dfrac{(x^2-3x+3)(1-x)}{(\sqrt[3]{3}x+\sqrt[3]{2})(\sqrt[3]{9}x^2-\sqrt[3]{6}x+\sqrt[3]{4})}.$

31) $f(x) = \dfrac{(2x^2+x-1)(x+1)}{x^2+x+4} = \dfrac{2\cdot(x-\frac{1}{2})\cdot(x+1)\cdot(x+1)}{x^2+x+4} = \dfrac{(2x-1)(x+1)^2}{x^2+x+4}$

$$(2x^2+x-1)\ :\quad x_{12} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm 3}{4} = \frac{-4}{4} = -1$$
$$= \frac{2}{4} = \frac{1}{2}$$

(x^2+x+4) nelze rozložit v $\mathbb{R}$, protože $D = \sqrt{1-16} < 0$.

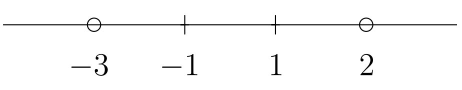
NB: $-1; \frac{1}{2} \quad \Rightarrow$



$\mathrm{D}(f) = \mathbb{R}$.

32) $f(x) = \dfrac{(x^2-1)(x+1)}{x^2+x-6} = \dfrac{(x-1)(x+1)(x+1)}{(x-2)(x+3)} = \dfrac{(x-1)(x+1)^2}{(x-2)(x+3)}$

NB: $-3; -1; 1; 2 \quad \Rightarrow$

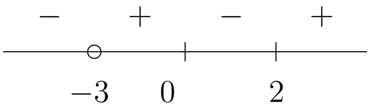


$\mathrm{D}(f) = \mathbb{R} - \{-3; 2\}$.

33) $f(x) = \dfrac{(x^4-3x^3)(x^2+2x+8)}{x+3} = \dfrac{x^3(x-3)\cdot(x^2+2x+8)}{x+3}$

(x^2+2x+8) nelze rozložit v $\mathbb{R}$, protože $D = \sqrt{4-32} < 0$.

NB: $-3; 0; 3 \quad \Rightarrow$



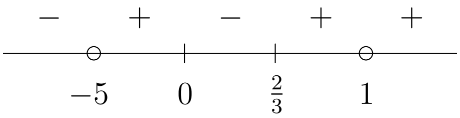
$\mathrm{D}(f) = \mathbb{R} - \{-3\}$.

34) $$f(x) = \frac{3x^3 - 5x^2 + 2x}{x^2 + 4x - 5} = \frac{(3x^2 - 5x + 2)x}{x^2 + 4x - 5} = \frac{3 \cdot (x - \frac{2}{2})(x-1)x}{(x-1)(x+5)} = \frac{(3x-2)(x-1)x}{(x-1)(x+5)} = \frac{(3x-2)x}{x+5}$$

$$(3x^2 - 5x + 2) : \quad x_{12} = \frac{5 \pm \sqrt{25 - 24}}{6} = \frac{5 \pm 1}{6} = 1$$

$$= \frac{4}{6} = \frac{2}{3}$$

NB: $-5; 0; \frac{2}{3}; 1 \quad \Rightarrow$

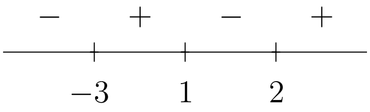


$\mathrm{D}(f) = \mathbb{R} - \{-5; 1\}$.

35) $$f(x) = \frac{(x^2 - 3x + 2)(x+3)}{x^2 + x + 1} = \frac{(x-2)(x-1)(x+3)}{x^2 + x + 1}$$

$(x^2 + x + 1)$ nelze rozložit v $\mathbb{R}$, protože $D = \sqrt{1 - 4} < 0$.

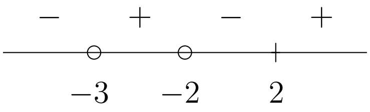
NB: $-3; 1; 2 \quad \Rightarrow$



$\mathrm{D}(f) = \mathbb{R}$.

36) $$f(x) = \frac{(x-2)(x^2+1)}{x^2 + 5x + 6} = \frac{(x-2)(x^2+1)}{(x+2)(x+3)}$$

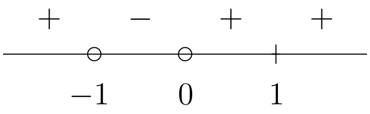
NB: $-3; -2; 2 \quad \Rightarrow$



$\mathrm{D}(f) = \mathbb{R} - \{-3; -2\}$.

37) $$f(x) = \frac{x - \frac{1}{x}}{x + \frac{3x+1}{x-1}} = \frac{\frac{x^2-1}{x}}{\frac{x^2-x+3x+1}{x-1}} = \frac{x^2-1}{x} \cdot \frac{x-1}{x^2+2x+1} = \frac{(x-1)^2(x+1)}{x(x+1)^2} = \frac{(x-1)^2}{x(x+1)}$$

NB: $-1; 0; 1 \quad \Rightarrow$



$\mathrm{D}(f) = \mathbb{R} - \{-1; 0\}$.

38) $f(x) = \ .$

Limita funkce

1) $$\lim_{x \to 1} \frac{x^2 - 1}{x^3 - x^2} = \left| \frac{0}{0} \right| = \lim_{x \to 1} \frac{(x-1)(x+1)}{x^2(x-1)} = \lim_{x \to 1} \frac{x+1}{x^2} = \frac{2}{1} = 2.$$

2) $\displaystyle\lim_{x\to3}\frac{x-3}{x^2-2x-3} = \left|\frac{0}{0}\right| = \lim_{x\to3}\frac{x-3}{(x-3)(x+1)} = \lim_{x\to3}\frac{1}{x+1} = \frac{1}{4}.$

3) $\displaystyle\lim_{x\to\infty}\frac{x^2+5x+4}{x^2-2x+3} = \left|\frac{\infty}{\infty-\infty}\right| = \lim_{x\to\infty}\frac{x^2(1+\frac{5}{x}+\frac{4}{x^2})}{x^2(1-\frac{2}{x}+\frac{3}{x^2})} = \left|\frac{1+0+0}{1-0+0}\right| = \frac{1}{1} = 1.$

4) $\displaystyle\lim_{x\to0}\frac{\sin x}{\sin(2x)} = \left|\frac{0}{0}\right| = \lim_{x\to0}\frac{\sin x}{2\cdot\sin x\cdot\cos x} = \lim_{x\to0}\frac{1}{2\cdot\cos x} = \frac{1}{2}.$

5) $\displaystyle\lim_{x\to-2}\frac{2x^2+5x+2}{x+2} = \left|\frac{0}{0}\right| = \lim_{x\to-2}\frac{(2x+1)(x+2)}{x+2} = \lim_{x\to-2}\frac{2x+1}{1} = -4+1 = -3.$

$$
\begin{aligned}
(2x^2+5x+2)\ :\quad x_{12} = \frac{-5\pm\sqrt{25-16}}{4} = \frac{-5\pm3}{4} &= \frac{-2}{4} = -\frac{1}{2}\\
&= \frac{-8}{4} = -2
\end{aligned}
$$

6) $\displaystyle\lim_{x\to\infty}\frac{2x^2+5x-1}{x^3-x^2+1} = \left|\frac{\infty}{\infty-\infty}\right| = \lim_{x\to\infty}\frac{x^3(\frac{2}{x}+\frac{5}{x^2}-\frac{1}{x^3})}{x^3(1-\frac{1}{x}+\frac{1}{x^3})} = \left|\frac{0+0+0}{1-0+0}\right| = \frac{0}{1} = 0.$

7) $\displaystyle\lim_{x\to-3}\frac{2x^2+3x-9}{x+3} = \left|\frac{0}{0}\right| = \lim_{x\to-3}\frac{2(x-\frac{3}{2})(x+3)}{x+3} = \lim_{x\to-3}\frac{2x-3}{1} = -6-3 = -9.$

$$
\begin{aligned}
(2x^2+3x-9)\ :\quad x_{12} = \frac{-3\pm\sqrt{9+72}}{4} = \frac{-3\pm9}{4} &= \frac{6}{4} = \frac{3}{2}\\
&= \frac{-12}{4} = -3
\end{aligned}
$$

8) $\displaystyle\lim_{x\to\infty}\frac{1-x}{x^2-1} = \left|\frac{1-\infty}{\infty-1}\right| = \left|\frac{-\infty}{\infty}\right| = \lim_{x\to\infty}\frac{1-x}{(x-1)(x+1)} = \lim_{x\to\infty}\frac{-1}{x+1} = \left|\frac{-1}{\infty}\right| = 0.$

9) $\displaystyle\lim_{x\to1}\frac{1-x}{x^2-1} = \left|\frac{0}{0}\right| = \lim_{x\to1}\frac{1-x}{(x-1)(x+1)} = \lim_{x\to1}\frac{-1}{x+1} = \frac{-1}{2}.$

NE na písemku:

*) $$\lim_{x\to-\infty}\left[x\left(\sqrt{x^2+1}+x\right)\right] = \left|-\infty\cdot0\right| = \lim_{x\to-\infty}\left[x\left(\sqrt{x^2+1}+x\right)\cdot\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x}\right] =$$
$$\lim_{x\to-\infty}\left(x\cdot\frac{x^2+1-x^2}{\sqrt{x^2+1}-x}\right) = \lim_{x\to-\infty}\left(x\cdot\frac{1}{\sqrt{x^2+1}-x}\right) = \lim_{x\to-\infty}\left(\frac{x}{|x|\cdot\sqrt{1+\frac{1}{x^2}}-x}\right) =$$
$$''\frac{-\infty}{+\infty-(-\infty)}'' = \lim_{x\to-\infty}\left(\frac{-1}{\sqrt{1+\frac{1}{x^2}}+1}\right) = \frac{-1}{\sqrt{1+0}+1} = -\frac{1}{1+1} = -\frac{1}{2}$$

Derivace

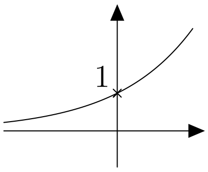
Zderivujte a určete $\mathrm{D}(f)$ a $\mathrm{D}(f')$:

1) $f(x) = (x-2)\sqrt{1-\mathrm{e}^x}$

$$f'(x) = 1\cdot\sqrt{1-\mathrm{e}^x} + (x-2)\cdot\frac{1}{2}\frac{-\mathrm{e}^x}{\sqrt{1-e^x}} = \frac{2(1-e^x)-x\cdot e^x+2e^x}{2\sqrt{1-e^x}} = \frac{2-2e^x+2e^x-x\cdot e^x}{2\sqrt{1-e^x}} =$$
$$= \frac{2-x\cdot e^x}{2\sqrt{1-e^x}}$$

D(f):

$$1-\mathrm{e}^x \geq 0$$
$$1 \geq \mathrm{e}^x$$
$$0 \geq x$$
$$\mathrm{D}(f) = \big(-\infty, 0\big\rangle$$



D(f'):

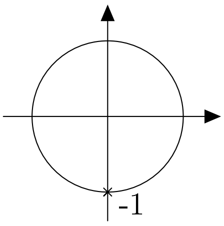
$$1-\mathrm{e}^x > 0$$
$$1 > \mathrm{e}^x$$
$$0 > x$$
$$\mathrm{D}(f') = (-\infty, 0)$$

2) $f(x) = \dfrac{\cos x}{3(1+\sin x)}$

$$f'(x) = \frac{1}{3}\left(\frac{-\sin x(1+\sin x) - \cos x(\cos x)}{(1+\sin x)^2}\right) = \frac{1}{3} \cdot \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \frac{1}{3} \cdot \frac{-(\sin x + 1)}{(1+\sin x)^2} =$$
$$= \frac{-1}{3(1+\sin x)}$$

D(f):

$$1 + \sin x \neq 0$$
$$\sin x \neq -1$$
$$x \neq \tfrac{3\pi}{2} + 2k\pi$$



$$\Rightarrow \qquad \mathrm{D}(f) = \mathrm{D}(f') = \mathbb{R} - \left\{\tfrac{3\pi}{2} + 2k\pi\right\}$$

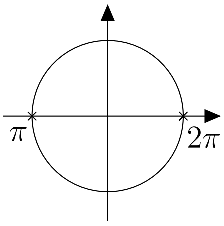
3) $f(x) = \mathrm{e}^{-2x} \cdot \cos(3x)$

$$f'(x) = \mathrm{e}^{-2x} \cdot (-2x) \cdot \cos(3x) + \mathrm{e}^{-2x} \cdot (-\sin(3x)) \cdot 3 = \mathrm{e}^{-2x}\left(-2\cos(3x) - 3\sin(3x)\right)$$

D(f) : $\quad \mathrm{e}^{-2x} = \frac{1}{e^{2x}} \quad \Rightarrow \quad e^{2x} \neq 0$ (platí vždy) $\quad \Rightarrow \quad \mathrm{D}(f) = \mathrm{D}(f') = \mathbb{R}$

4) $f(x) = \dfrac{-\cos x}{2\sin^2 x}$

$$f'(x) = \frac{\sin x \cdot \sin^2 x + \cos x \cdot 2 \cdot 2 \cdot \sin x \cdot \cos x}{4\sin^4 x} = \frac{2 \cdot \sin x\left(\sin^2 x + 2\cos^2 x\right)}{4 \cdot \sin^4 x} =$$
$$= \frac{\sin^2 x + \cos^2 x + \cos^2 x}{2 \cdot \sin^3 x} = \frac{1 + \cos^2 x}{2\sin^3 x}$$



$$\Rightarrow \qquad \mathrm{D}(f) = \mathrm{D}(f') = \mathbb{R} - \{k\pi\}$$

5) $f(x) = \log_2 \sqrt{1-x^2} + \frac{1}{x}$

$$f'(x) = \frac{1}{\sqrt{1-x^2} \cdot \ln 2} \cdot \frac{1}{2} \cdot \frac{-2x}{\sqrt{1-x^2}} + \frac{-1}{x^2} = \frac{-x}{(1-x^2)\ln 2} - \frac{1}{x^2}$$

D(f): $\quad 1 - x^2 > 0 \quad \wedge \quad x \neq 0 \quad \wedge \quad \sqrt{1-x^2} > 0$ (platí vždy)

$$1 > x^2$$
$$x \in (-1; 1)$$

D(f') : $x \neq \pm 1$

$$x \neq 0$$
$$\mathrm{D}(f') = \mathbb{R} - \{0; \pm 1\}$$

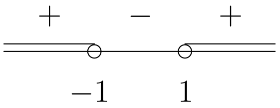
$$\mathrm{D}(f) = (-1; 1) - \{0\} = \mathrm{D}(f')$$

6) $f(x) = \ln \dfrac{x+1}{x-1}$

$$f'(x) = \frac{x-1}{x+1} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{x-1-x-1}{(x+1)(x-1)} = \frac{-2}{x^2-1}$$

$\mathrm{D}(f): \quad \frac{x+1}{x-1} > 0 \quad \wedge \quad x-1 \neq 0 \qquad\qquad \mathrm{D}(f'): x \neq \pm 1$

$x \neq 1$



$\Rightarrow \quad \mathrm{D}(f) = (-\infty; -1) \cup (1; \infty) = \mathrm{D}(f')$

7) $f(x) = \dfrac{x^3 \cdot \sin x}{\cos x - 1}$

$$f'(x) = \frac{(3x^2 \cdot \sin x + x^3 \cdot \cos x) \cdot (\cos x - 1) + x^3 \cdot \sin^2 x}{(\cos x - 1)} =$$

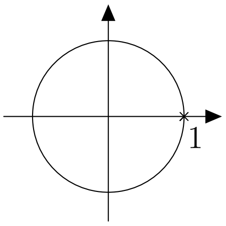
$$= \frac{3x^2 \cdot \sin x \cos x - 3x^2 \cdot \sin x + x^3 \cdot \cos^2 x - x^3 \cdot \cos x + x^3 \cdot \sin^2 x}{(\cos x - 1)^2} =$$

$$= \frac{3x^2 \cdot \sin x(\cos x - 1) + x^3(\cos^2 x + \sin^2 x) - x^3 \cos x}{(\cos x - 1)^2} =$$

$$= \frac{3x^2 \cdot \sin x(\cos x - 1) + x^3(1 - \cos x)}{(\cos x - 1)^2} = \frac{(\cos x - 1)(3x^2 \sin x - x^3)}{(\cos x - 1)^2} = \frac{x^2(3 \sin x - x)}{\cos x - 1}$$

$\mathrm{D}(f)$: $\quad \cos x \neq 1$

$x \neq k\pi$



$\Rightarrow \quad \mathrm{D}(f) = \mathrm{D}(f') = \mathbb{R} - \{k\pi\}$

8) $f(x) = \log \sqrt{1 - x^2} = \log_{10} \sqrt{1 - x^2}$

$$f'(x) = \frac{1}{\sqrt{1-x^2} \cdot \ln 10} \cdot \frac{1}{2} \cdot \frac{-2x}{\sqrt{1-x^2}} = \frac{-x}{(1-x^2)\ln 10}$$

$\mathrm{D}(f)$: $\quad 1 - x^2 > 0 \quad \wedge \quad \sqrt{1-x^2} > 0$ (platí vždy) $\qquad \mathrm{D}(f'): 1 \neq x^2$

$1 > x^2 \qquad\qquad x \neq \pm 1$

$x \in (-1; 1) \qquad\qquad \mathrm{D}(f') = \mathbb{R} - \{\pm 1\}$

$\mathrm{D}(f) = (-1; 1) = \mathrm{D}(f')$

9) $f(x) = \sqrt{1 + \left(\dfrac{x}{\sqrt{1-x^2}}\right)^2} = \sqrt{1 + \dfrac{x^2}{1-x^2}} = \sqrt{\dfrac{1 - x^2 + x^2}{1-x^2}} = \dfrac{1}{\sqrt{1-x^2}} = (1-x^2)^{-\frac{1}{2}}$

$$f'(x) = -\frac{1}{2} \cdot (1-x^2)^{-\frac{3}{2}} \cdot (-2x) = \frac{x}{\sqrt{1-x^2}^3}$$

$\mathrm{D}(f): \; 1 - x^2 > 0$

$1 > x^2 \qquad \Rightarrow \qquad \mathrm{D}(f) = (-1; 1).$

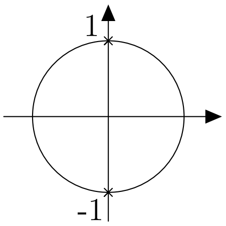
10) $f(x) = \dfrac{\sin x + 1}{\cos x}$

$$f'(x) = \frac{\cos x \cdot \cos x - (\sin x + 1)(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1 + \sin x}{1 - \sin^2 x} =$$

$$= \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)} = \frac{1}{1 - \sin x}$$

D(f): $\quad \cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi$



$\Rightarrow \quad \mathrm{D}(f) = \mathrm{D}(f') = \mathbb{R} - \left\{\frac{\pi}{2} + k\pi\right\}$

Tečna ke grafu funkce

1) $T[0,8], \quad t : y = 4x + 8, \quad n : x + 4y - 32 = 0.$

2) $T[1, \frac{\pi}{4}], \quad t : y = \frac{1}{2}x + \frac{\pi}{4} - \frac{1}{2}, \quad n : y = -2x + \frac{\pi}{4} + 2.$

3) $T[\frac{\pi}{6}, 2], \quad t : y = 2x + 2 - \frac{\pi}{2}, \quad n : 2y = -x + 4 + \frac{\pi}{4}.$

4) $T[1,1], \quad t : y = 2x - 1, \quad n : y = -\frac{1}{2}x + \frac{3}{2}.$

5) $T[3,9], \quad t :, \quad n :.$

6) $T[4,24], \quad t : y = 10x - 16, \quad n : y = -\frac{1}{10}x + \frac{122}{5}.$

7) $T[1, \frac{1}{2}], \quad t : x + 2y - 2 = 0, \quad n : 4x - 2y - 3 = 0.$

8) $f(x) = \mathrm{e}^{2x},\ f(0) = \mathrm{e}^0 = 1 \quad \Rightarrow \quad T[0,1]$

$f'(x) = 2\mathrm{e}^{2x} \quad \Rightarrow \quad k = f'(0) = 2 \cdot \mathrm{e}^0 = 2$

$$\begin{aligned} t &: y = 2x + q \\ T \in t &: 1 = 0 + q \ \Rightarrow \ q = 1 \\ t &: y = 2x + 1 \end{aligned}$$

$$\begin{aligned} n &: y = -\frac{1}{2}x + q \\ T \in n &: 1 = 0 + q \ \Rightarrow \ q = 1 \\ n &: y = -\frac{1}{2}x + 1 \\ & 2y = -x + 2 \\ & x + 2y - 2 = 0 \end{aligned}$$

9) $f(x) = \ln x,\ f(\mathrm{e}) = \ln \mathrm{e} = 1 \quad \Rightarrow \quad T[\mathrm{e}, 1]$

$f'(x) = \dfrac{1}{x} \quad \Rightarrow \quad k = f'(\mathrm{e}) = \dfrac{1}{\mathrm{e}}$

$$\begin{aligned} t &: y = \frac{1}{\mathrm{e}}x + q \\ T \in t &: 1 = \frac{1}{\mathrm{e}}\mathrm{e} + q \\ & 1 = 1 + q \ \Rightarrow \ q = 0 \\ t &: y = \frac{1}{\mathrm{e}}x \\ & \mathrm{e}y = x \\ & x - \mathrm{e}y = 0 \end{aligned}$$

$$\begin{aligned} n &: y = -\mathrm{e}x + q \\ T \in n &: 1 = -\mathrm{e} \cdot \mathrm{e} + q \ \Rightarrow \ q = 1 + \mathrm{e}^2 \\ n &: y = -\mathrm{e}x + 1 + \mathrm{e}^2 \end{aligned}$$

Interpolační polynom

1) $P(x) = -\frac{5}{4}x^3 - \frac{1}{6}x^2 + \frac{13}{4}x + \frac{1}{6}.$

2) $P(x) = -\frac{1}{2}x^3 - \frac{2}{3}x^2 + \frac{1}{2}x + \frac{8}{3}.$

3) $P(x) = -\frac{2}{3}x^3 - \frac{1}{6}x^2 + 5x - \frac{4}{3}$.

4) $P(x) = -x^3 + 3x$.

5) $P(x) = \frac{1}{10}x^3 + \frac{11}{15}x^2 + \frac{1}{10}x - \frac{44}{15}$.

6) $P(x) = -\frac{1}{10}x^3 - \frac{5}{21}x^2 + \frac{29}{10}x + \frac{125}{21}$.

7) $P(x) = -\frac{1}{12}x^3 + \frac{1}{3}x^2 - \frac{1}{2}x + 2$.

8) $P(x) = \frac{1}{15}x^3 + \frac{1}{5}x^2 - \frac{1}{15}x + \frac{9}{5}$.

9) $P(x) = -\frac{1}{8}x^3 - \frac{3}{8}x^2 + \frac{1}{8}x + \frac{19}{8}$.

10) $P(x) = -\frac{1}{2}x^3 - 3x^2 - \frac{5}{2}x + 2$.

11) $P(x) = \frac{1}{4}x^3 - \frac{25}{4}x + 2$.

12) $P(x) = -x^3 - 6x^2 - 5x + 2$.

13) $P(x) = \frac{2}{5}x^3 - \frac{9}{5}x^2 - \frac{11}{5}x + 2$.

14) $P(x) = -x^3 + 5x + 2$.

15) $P(x) = -\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 2$.

16) $P(x) = 6x^3 + 15x^2 + 9x + 2$.