Some examples of generated fuzzy implicators

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Conjunctors in MV-logic with truth values range $[0,1]$ are monotone extensions of the classical conjunction. Let $f : [0,1] \to [0,\infty]$ be a strictly decreasing function, such that $f(1) = 0$, then we can define conjunctor $C : [0,1]^2 \to [0,1]$ by

$$C(x,y) = f^{(-1)}(f(x) + f(y)),$$

where the pseudo-inverse $f^{(-1)}$ is given by $f^{(-1)}(x) = \sup\{t \in [0,1]; f(t) > x\}$, f is called an additive generator of C. A function $I : [0,1]^2 \to [0,1]$ is said to an implicator if and only if $I(1,0) = 0$ and $I(0,0) = I(0,1) = I(1,1) = 1$ and I is non-increasing in its first component and non-decreasing in its second component. The unary operator $n : [0,1] \to [0,1]$ is called negator if for any $a, b \in [0,1]$ holds

$$a \leq b \Rightarrow n(b) \leq n(a),$$

$$n(0) = 1, n(1) = 0.$$

Starting with the conjunctor C and standard negation $N_s(x) = 1 - x$, we can introduce the implication operator in $[0,1]-$ valued logic as follows: $I_C(x,y) = 1 - C(x, 1-y)$. Another way of extending the classical binary implication operator to the unit interval $[0,1]$ uses the *residuation* R_C with respect to the left-continuous conjunctor C

$$R_C(x,y) = \sup\{z \in [0,1]; C(x,z) \leq y\}.$$

There exists several constructions of implicators. We will compare these implicators and some their properties will be given.

References

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