The solution of differential equation of physical problem by the use of Matlab

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Abstract

At the present time teachers try to introduce and use information technologies in educational process of technical subjects at schools as mathematics, physics, chemistry. Apart from e-learning support (electronic materials and textbooks available on the internet), there are various mathematical software environments as Matlab, Mathematica, MS Excel etc. The paper points out the possibility of Matlab utilization at solving differential equation of physical problem.

1. Introduction

Physical problem provides knowledge in various forms – as a means of motivation, information about examined phenomenon, but also as a set of functions needed for the solution of the problem. A student – solutionist of physical problem – has to carry out a certain program consisting of partial functions - steps [1]. Creation of mathematical models of physical phenomena is an important part of physical problems solution. Modelling is one of theoretical cognitive methods whose characteristic feature is cognitive process, in which examined phenomenon – an original cognitive object – is replaced by a model cognitive object – a model. Under the term of mathematical model is understood a sum of mathematical relations capable of quantitative description of physical phenomenon. Mathematical model is an abstraction and idealization of real phenomenon [2]. Model has to be created as a functional one. Functionality of model is estimated according to its behavior at the change of conditions, at which it works. The form of mathematical model depends on used mathematical tools. Mathematical model of physical problem solved within the subject of Physics of bachelor´s studies at the Faculty of Manufacturing Technologies of the Technical University of Košice with a seat in Prešov is presented in the following part of the paper.

2. The solution of physical problem

Assignment: Steel cannonball with the r radius and ρ density is fired with the initial velocity v_0 from the cannon of the Coast Artillery under an α angle. Determine the length of cannonball firing range. Solve the problem for numerical values: $\alpha = 45^o$, $r = 0,05m$, $\rho = 7800 kg.m^{-3}$, $v_0 = 1000 m.s^{-1}$, $c = 0,26 kg.m^{-1}$.

Solution: The solution of the problem is based on the fact that the cannonball carries out curvilinear motion in the gravitational field of the Earth that is called projection at an angle. It

is the motion consisting of two motions: uniform motion and vertical throw upward with the initial velocity v_0 (fig. 1).
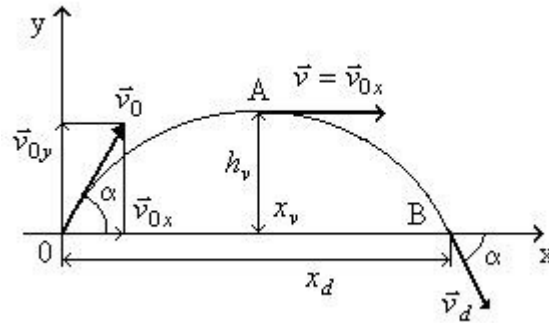

Figure 1. The motion of cannonball.

We proceed from the balance of forces to make out equations. For the balance of forces at the direction of x axis is valid: $F_x + F_{ox} = 0 \Leftrightarrow m\dfrac{d^2x}{dt^2} + cS\left(\dfrac{dx}{dt}\right)^2 = 0$, where F_x is the power of acceleration at the direction of x axis according to the Newton´s second law, F_{ox} is the resistance power at the direction of x axis, c is the coefficient of aerodynamic resistance of the cannonball, S is the frontal area of the shot. For the balance of forces at the direction of y axis is valid: $F_y + F_{oy} + F_g = 0 \Leftrightarrow m\dfrac{d^2y}{dt^2} + cS\left(\dfrac{dy}{dt}\right)^2 = -mg$, where F_y is the power of acceleration at the direction of y axis according to the Newton´s second law, F_g is the gravitation force, F_{oy} is the resistance power at the direction of y axis, c is the coefficient of aerodynamic resistance of the cannonball, S is the frontal area of the shot.

The system of two differential equations of the second order without or with the second member was obtained:

$$m\dfrac{d^2x}{dt^2} + cS\left(\dfrac{dx}{dt}\right)^2 = 0 \qquad\qquad m\dfrac{d^2y}{dt^2} + cS\left(\dfrac{dy}{dt}\right)^2 = -mg \qquad\qquad (1)$$

In case the resistance of environment is not considered (i.e. the coefficient of aerodynamic resistance $c = 0$), we have the system of equations for the balance of forces at the direction of x and y axes: $m\dfrac{d^2x}{dt^2} = 0$ and $m\dfrac{d^2y}{dt^2} = -mg$. By their solution and integration concerning the initial conditions $y(t=0) = 0$ we obtain the quadratic equation: $-x^2\dfrac{g}{2v_0^2\cos^2\alpha} + x.tg\alpha = 0$ [3], from which results that trajectory of motion of the cannonball has the form of parabola. The solution of the quadratic equation is searched length of cannonball firing range $x = 101\ 936{,}79 m$.

If the air resistance is not neglected at the solution of the physical problem, it is necessary to solve the system of differential equations of the second order (1). If the relations for the weight, volume and frontal area of the cannonball $m = \rho V$, $V = \dfrac{4}{3}\pi r^3$, $S = \pi r^2$ are used, we

introduce the constant $k = c\dfrac{3}{4\rho r}$ and denote $x = h_x$ a $y = h_y$, the relations (1) will be transposed into the form of:
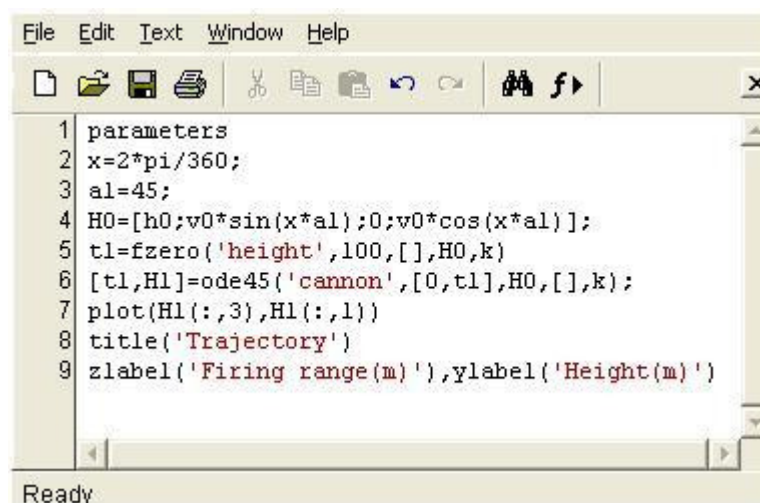
$$\frac{d^2 h_x}{dt^2} + k\left(\frac{dh_x}{dt}\right)^2 = 0 \qquad h_x(t = 0) = x_0 \qquad v_x = \frac{dh_x}{dt}\bigg|_{t=0} = v_0\cos\alpha$$

$$\frac{d^2 h_y}{dt^2} + k\left(\frac{dh_y}{dt}\right)^2 = -g \qquad h_y(t = 0) = y_0 \qquad v_y = \frac{dh_y}{dt}\bigg|_{t=0} = v_0\sin\alpha$$

(2)

The system of two differential equations of the second order without or with the right side of equation was obtained. An analytic solution of the problem is complex and it does not lead to such simple dependancies for the coordinates of the ball shot as it was in case the air resistance was not considered. The problem will thus be solved numerically by using an interactive program Matlab.

3. MATLAB utilization at solving physical problem

Matlab presents a highly efficient language for technical calculations. It connects calculations, visualization and programming into simply operable environment [4]. It is an interactive device, where the basic data type is the field without necessity to declare its measurements. This feature together with the number of built-in functions enables relatively easy solution of many technical problems. It is a standard tool at teaching at school, for example mathematics and other technical subjects, but also an effective tool for research, development and analysis of data.

One of the methods of solution in Matlab is the m-files formation, which were used at solving the problem. We created an m-file, into which we transformed the system of differential equations of the second order (2) concerning the initial conditions into the system of equations of the first order in the form of matrices. Then we formed a script, in which the parameters used at solving the problem were defined. Single solution consists of the calculation of length and time of the cannonball firing range and portrayal of the trajectory. M-file *strela2.m* for the portrayal of the distance of the cannonball firing range is created in fig. 2.


```
File  Edit  Text  Window  Help

1  parameters
2  x=2*pi/360;
3  al=45;
4  H0=[h0;v0*sin(x*al);0;v0*cos(x*al)];
5  tl=fzero('height',100,[],H0,k)
6  [tl,Hl]=ode45('cannon',[0,tl],H0,[],k);
7  plot(Hl(:,3),Hl(:,1))
8  title('Trajectory')
9  zlabel('Firing range(m)'),ylabel('Height(m)')

Ready
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Figure 2. M-file for depiction of the graph.

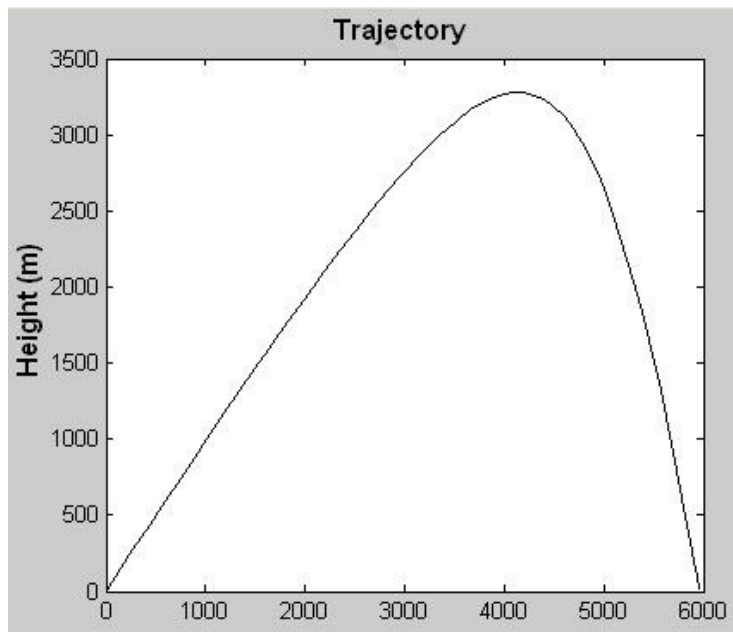The graph of trajectory of the cannonball at given α angle depicted by created m-file is presented in fig. 3.

Figure 3. Graph of trajectory.

Single solution of the problem presents launching the script (fig. 4).

Figure 4. M-file for the calculation of the length and time of firing range.

It results from the numerical solution of such formed mathematical model of the physical problem that firing range of the cannonball in case the air resistance is considered is $x = 5\ 960,3m$.

4. Conclusions

It emerges from the results of solving the physical problem that the results obtained at solving the problem differ concerning the formed mathematical model. Mathematical model of the problem, if the air resistance was considered, led to more complex relations and numerical method of solving by Matlab was used. Introduction and utilization of information and communication means make teaching more attractive, which is necessary for the subjects as mathematics, physics, chemistry, because they occur among the least favourite subjects.

References

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