Two examples of the constructions of non-continuous t-norms

Dana Hliněná
Department of Mathematics, FEEC, Brno University of Technolog
Technická 8, 616 00 Brno, Czech Rep.
e-mail: hlinena@feec.vutbr.cz

In this contribution we present some interesting constructions of non-continuous triangular norms.

The first approach is based on strictly increasing sequences of natural numbers. An associative commutative monotone and bounded by minimum binary operation on these sequences induces a t-norm. We discuss a t-norm based on the original idea of Budinčevič and Kurilič [1].

Example 1 *For* $(x, y) \in]0,1]^2$ *let*

$$x = \sum_{i=1}^{\infty} \frac{1}{2^{x_i}} \text{ and } y = \sum_{i=1}^{\infty} \frac{1}{2^{y_i}},$$

be the unique dyadic representations of x *and* y*. Then the t-norm* $T_1 : [0,1]^2 \to [0,1]$ *is given by*

$$T_1(x, y) = \begin{cases} \sum\limits_{i=1}^{\infty} \frac{1}{2^{x_i+y_i}}, & \text{if } (x, y) \in]0,1[^2, \\ \min(x, y), & \text{otherwise,} \end{cases}$$

and the t-norm $T_2 : [0,1]^2 \to [0,1]$ *is given by*

$$T_2(x, y) = \begin{cases} \sum\limits_{i=1}^{\infty} \frac{1}{2^{x_i \cdot y_i}}, & \text{if } (x, y) \in]0,1[^2, \\ \min(x, y), & \text{otherwise.} \end{cases}$$

Both T_1 *and* T_2 *are Archimedean and strictly monotone t-norms, which are neither left nor right continuous.*

The usual requirement on a t-norm T in fuzzy logic to model a conjuction is its left-continuity. Then, implication can be modeled by the corresponding residual operator. Therefore we will investigate t-norms with similar properties as in [1] under additional requirement of their left-continuity.

Several other interesting properties of this t-norm are investigated, including its residual implicator.

Second approach uses idea of multiplicative generator φ of a triangular norm. Multiplicative generator φ of a triangular norm is a strictly increasing function $\varphi : [0,1] \to [0,1]$ such that $\varphi(1) = 1$ and $\varphi(x).\varphi(y) \in H(\varphi)$ or $\varphi(x).\varphi(y) < \varphi(0)$. The corresponding t-norm T is defined by means of φ as follows:

$$T(x, y) = \varphi^{(-1)}(\varphi(x).\varphi(y)),$$

where $\varphi^{(-1)} : [0,1] \to [0,1]$ is a so-called pseudo-inverse of φ defined by

$$\varphi^{(-1)}(t) = \sup(x \in [0,1]; \varphi(x) < t)$$

with convention $\sup \emptyset = 0$.

If strictly increasing function φ is left continuous, but non-continuous, then associativity of induces operator is violated, [6]. However, then the operation

$$T_*(x,y) = \begin{cases} \min(x,y) & \text{if } \max(x,y) = 1, \\ \varphi(\varphi^{(-1)}(x).\varphi^{(-1)}(y)) & \text{otherwise,} \end{cases}$$

defines a non-continuous t-norm. Some example will be given.

References

[1] M. Budinčevič, M. Kurilič . A family of strict and discontinuous triangular norms. *Fuzzy Sets and Systems*, **95**: 381-384, 1998

[2] E.P.Klement, R.Mesiar. Triangular norms:a tutorial. *Tatra Mountains Math. Publ.*, **13**: 169-193, 1997.

[3] R. Mesiar, V. Novák. Open problems from the 2nd International Conference on Fuzzy Sets Theory and Its Applications. *Fuzzy Sets and Systems*, **81**: 185-190, 1996

[4] E.P.Klement, R.Mesiar, E.Pap. Additive generators of t-norms which are not necessarily continuous. *Proc. EUFIT'96, Aachen*: 70-73, 1996

[5] C.M.Ling. Representation of associative functions. *Publ. Math. Debrecen*, **12**: 189-212, 1965

[6] B.Schweizer, A.Sklar. Probabilistic Metric Spaces *North Holland, New York*, 1983

[7] D. Smutná. On a peculiar t-norm. *Busefal*, **75**: 60-67, 1998

[8] D. Smutná. A note on non-contiuous t-norms. *Busefal*, **76**: 19-24, 1998