On the two-scale finite element method for the heat transfer in buildings

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The design of modern engineered buildings cannot avoid understanding non-stationary thermal behaviour of buildings, its prediction and modification. of the design of modern engineered buildings. Reliable mathematical models and corresponding software codes should contain information about a (quasi)periodic material microstructure. One possible approach is to apply the correction algorithm for (not necessarily conforming) two-scale finite element discretization (Glowinski et al. 2004) to a special parabolic problem (using the sequences of Rothe) and to combine it with the two-scale homogenization theory (Nguetseng 1989, Allaire 1992, Holmbom 1995, Cioranescu & Donato 1999, etc.).

Let Ω be a domain in R^3, $V = W^{1,2}(\Omega)$ and $I = [0,T]$ (a time interval). The most simple technically realistic problem of heat transfer is to find such temperature $\tau \in L^2(I,V)$ with a time derivative $\dot{\tau} \in L^2(I,V^*)$ that, in terms of scalar products $(.,.)$ in $L^2(\Omega)$ and $\langle .,. \rangle$ in $L^2(\partial\Omega)$,

$$(v, \widetilde{c}\dot{\tau}) + (\nabla v, \widetilde{a}\nabla\tau) + \langle v, b\tau \rangle = \langle v, b\tau^{\times} \rangle \;\forall\, v \in V\,;$$

some additional assumptions related to material characteristics a, b, c are needed, $\widetilde{\ }$ refers to some "effective values" (rarely known a priori), the temperature of outer environment $\tau^{\times}$ and initial values of τ in zero time are prescribed. The discretized forms for two (macro- and micro-) scales are

$$(v_h, \widetilde{c}(\tau_{sh} - \tau_{s-1\,h}))/H + (\nabla v_h, \widetilde{a}\nabla\tau_{sh}) + \langle v_h, b\tau_{sh} \rangle = \langle v_h, b\tau^{\times}_{sh} \rangle \;\forall\, v_h \in V_h\,,$$

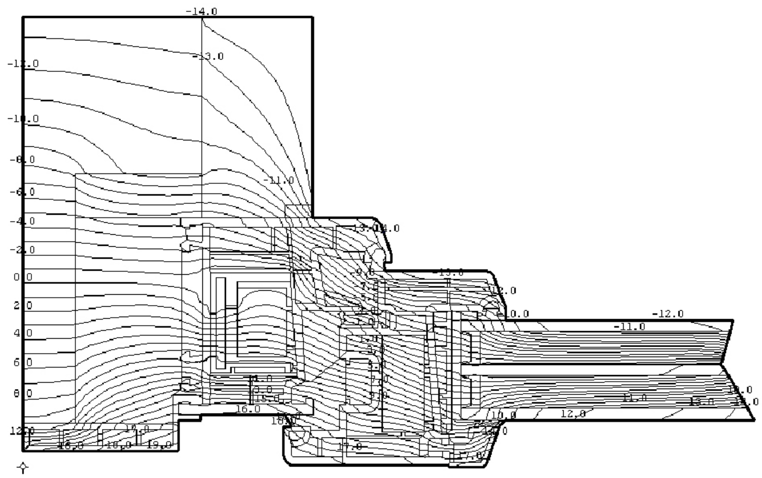
$$(v_\delta, c(./\varepsilon)(\tau^{\varepsilon}_{s\delta} - \tau^{\varepsilon}_{s-1\,\delta}))/H + (\nabla v_\delta, a(./\varepsilon)\nabla\tau^{\varepsilon}_{s\delta}) + \langle v_\delta, b\tau^{\varepsilon}_{s\delta} \rangle = \langle v_\delta, b\tau^{\times}_{s\delta} \rangle \;\forall\, v_\delta \in V_\delta$$

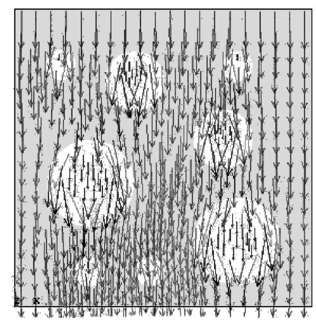
where $h, H, \delta \to 0$, $h \gg \delta$, V_h and V_δ are finite-dimensional subspaces of V, $s \in \{1, 2, \ldots, T/H\}$, ε is a characteristic microstructural size ($\varepsilon \to 0$ during homogenization) and a, c are periodic on certain parallelipeds in R^3. The convergence properties of a special iterative process can be compared with classical finite element techniques.

The full version CD-ROM version of this paper includes:

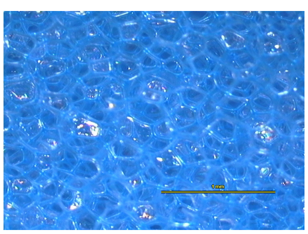
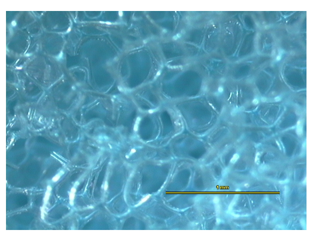
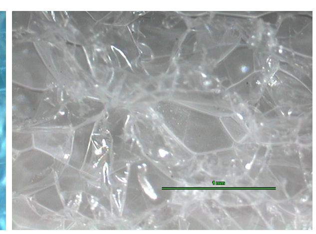
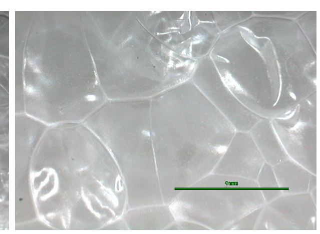
1. overview of modelling of the heat transfer in buildings,
2. analysis of two-scale grids and two-scale homogenization,
3. description of an iterative computational algorithm for the stationary heat transfer.

Two upper illustrative figures show i) temperature isotherms in a part of certain advanced (macroscopic) window construction, ii) distributions of heat fluxes $-A\nabla\tau$ in a non-homogenous rubber-based insulation layer in great detail (a square with edge length 0.1 mm). Our original stationary problem in R^3 is reduced to the two-dimensional one, all computations are ANSYS-supported.





The lower sequence of figures demonstrates some typical examples of microstructure of real insulation materials.

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