On Some Applications of Triangular Norms

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1 Introduction

The aim of this paper is to give a sound resolution deduction rule in many-valued logic with truth value range $[0, 1]$, and with arbitrary connectives and graded information. We introduce and study the resolution truth function $f_\vee(x, y)$ such that

$$\frac{(\gamma \vee \alpha, x), (\beta \vee \neg\alpha, y)}{(\gamma \vee \beta, f_\vee(x, y))}$$

is a sound rule, and $f_\vee(x, y)$ gives the biggest possible values for which the rule is sound. We will study existence and properties of resolution truth functions in different logical systems.

Fuzzy resolution (or also many-valued resolution) was already studied to handle uncertain or partially erroneous information by D. Mundici and N. Olivetti in [3] for Łukasiewicz logic with distinguished disjunction used for building clauses being the Gödel disjunction max. Moreover they were restricted to the case when $x = y$, that is lower bounds of truth degree of both clauses are the same. There is also a different family of resolution rules in possibilistic logic. Although they look similar, they have radically different semantics.

In this contribution we deal with the truth function for negation as $n(x) = 1 - x$. Conjunctors and disjunctors in MV-logic with truth value range $[0, 1]$ are monotone extensions of the classical conjunction and disjunction. Commonly used conjunctors in MV-logic are the triangular norms.

Definition. A *triangular norm* (*t-norm* for short) is a binary operation on the unit interval $[0, 1]$, i.e., a function $T : [0, 1]^2 \to [0, 1]$, such that the following four axioms are satisfied for all $x, y, z \in [0, 1]$:
(T1) *Commutativity* $T(x, y) = T(y, x)$,
(T2) *Associativity* $T(x, T(y, z)) = T(T(x, y), z)$,
(T3) *Monotonicity* $T(x, y) \leq T(x, z)$ whenever $y \leq z$,
(T4) *Boundary Condition* $T(x, 1) = x$.

The dual operator to the t-norm T is the triangular conorm (t-conorm), modeling a disjunction, which is given by $S(x, y) = 1 - T(1 - x, 1 - y)$.

2 The aggregation deficits

First we introduce a new operator, let us call it an *aggregation deficit* R_D, which is based on a disjunctor D. Having in mind our task of sound resolution described in the Introduction, the motivation is the following: Assume the truth value $TV(\alpha) = a$. We would like to know conditions on truth values $TV(\beta) = b$ and $TV(\gamma) = c$ such that they aggregate together with a ($1 - a$ respectively) to have $D(c, a) \geq x$ and $D(b, 1 - a) \geq y$. Having this we will be able to give estimates on the truth value of $\gamma \vee \beta$ which does not depend on a.

In other words we look for an aggregation deficit R_D such that following holds

$$\begin{array}{rcl} x \leq D(c, a) & \text{iff} & c \geq R_D(a, x), \\ y \leq D(b, 1 - a) & \text{iff} & b \geq R_D(1 - a, y). \end{array}$$

This leads naturally to the following definition:

Definition. Let the aggregation deficit $R_D(a, t) = \inf(z \in [0, 1]; D(z, a) \geq t)$.

The properties of aggregation deficits related to specific t-conorms will be given in the poster.

3 The resolution truth function

For the formulation of a result on sound and complete full resolution (for partial results see [3]), we will investigate the resolution truth function which is defined by the following definition.

Definition. Assume D is a disjunctor and R_D is the corresponding aggregation deficit. The *resolution truth function* $f_{R_D} : [0, 1]^2 \to [0, 1]$ is defined by

$$f_{R_D}(x, y) = \inf_{a \in [0,1]} (D(R_D(a, x), R_D(1 - a, y))).$$

This definition is related to a construction of non-commutative conjunctions obtained by reconstructing a conjunction from a residuated implication, see [1].

In the poster, we introduce the resolution truth function which enables us to define a sound graded many-valued resolution. We study its properties from the point of view of fuzzy operators. Conditions under which this function yields an aggregation operator are given. We give some explicit formulas for resolution truth functions generated by specific conorms.

References

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