

Příklad: Najděte interpolační polynom funkce f užitím hodnot $y_i = f(x_i)$ v uzlech x_i z tabulky.

a) Lagrangeův tvar, b) Newtonův tvar

i	1	2	3	4
x_i	-1	0	1	2
y_i	-4	-1	0	5

$$a) L = y_1 L_1 + y_2 L_2 + y_3 L_3 + y_4 L_4$$

$$L_1 = \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} = \frac{x^3 - 3x^2 + 2x}{-6}$$

$$L_2 = \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} = \frac{x^3 - x - 2x^2 + 2}{2}$$

$$L_3 = \frac{(x+1)(x-0)(x-2)}{(1+1)(1-0)(1-2)} = \frac{x^3 - x^2 - 2x}{-2}$$

$$L_4 = \frac{(x+1)(x-0)(x-1)}{(2+1)(2-0)(2-1)} = \frac{x^3 - x}{6}$$

$$L = 4 \cdot \frac{x^3 - 3x^2 + 2x}{-6} - 1 \cdot \frac{x^3 - 2x^2 - x + 2}{2} + 0 \cdot \frac{x^3 - x^2 - 2x}{-2} + 5 \cdot \frac{x^3 - x}{6} =$$

$$= \frac{4x^3 - 12x^2 + 8x - 3x^3 + 6x^2 + 3x - 6 + 5x^3 - 5x}{6} = \underline{\underline{x^3 - x^2 + x - 1}}$$

h)

	x_i	y_i	
x_1	-1	-4	
x_2	0	-1	$y_{21} = \frac{-1 - (-4)}{0 - (-1)} = 3$
x_3	1	0	$y_{32} = \frac{0 - (-1)}{1 - 0} = 1$
x_4	2	5	$y_{43} = \frac{5 - 0}{2 - 1} = 5$

Additional calculations shown in the image:

- $y_{321} = \frac{1 - 3}{1 - (-1)} = -1$
- $y_{432} = \frac{5 - 1}{2 - 0} = 2$
- $y_{4321} = \frac{2 - (-1)}{2 - (-1)} = 1$

$$N(x) = y_1 + y_{21}(x-x_1) + y_{321}(x-x_1)(x-x_2) + y_{4321}(x-x_1)(x-x_2)(x-x_3)$$

$$N(x) = -4 + 3(x+1) + (-1)(x+1)(x-0) + 1(x+1)(x-0)(x-1) =$$

$$= -4 + 3x + 3 - x^2 - x + x^3 - x = \underline{\underline{x^3 - x^2 + x - 1}}$$