STABILITY OF FREDHOLM'S INTEGRO-DIFFERENTIAL EQUATIONS

ZDENĚK ŠMARDA

Abstract: This paper deals with stability Fredholm's integro-differential equations and associated ordinary differential equations.

Key words: Integro-differential equation, stability of solution, asymptotic stability of solution.

AMS Subject Classification: 45J05

In this paper there are investigated classes of systems of Fredholm's integro-differential equations in which Fredholm's integral terms will change the asymptotic behaviour of solutions of associated systems of ordinary differential equations.

At the first we consider the following elementary initial value problem:

$$u'(t) + u(t) = \lambda \int_0^1 u(s)ds, \quad u(0) = u_0, \tag{1}$$

where $u_0,\ \lambda \in R,\ t \in J = [0, \infty)$.
This problem has the solution

$$u(t) = \left[e^{-t}(e - \lambda e) + \lambda e - \lambda\right] \frac{u_0}{e - \lambda}.$$

From this it follows that for $\lambda \neq e$ the trivial solution of (1) is only stable but the trivial solution of the associated differential equation

$$u' + u = 0$$

is asymptotic stable.

Now we consider the following problem

$$u'(t) = Au(t) + \lambda f\left(t, u(t), \int_0^1 K(t, s, u(s))ds\right), \quad u(0) = u_0, \tag{2}$$

where $f \in C^0(\Omega_0, R^n),\ \Omega_0 = \{(t, u, v) \in J \times R^n \times R\},\ J = [0, \infty),\ f(t, 0, 0) \equiv 0$, $K \in C^0(\Omega_1, R^n),\ \Omega_1 = \{(t, s, u) \in J \times J_0 \times R^n\},\ J_0 = [0, 1],\ K(t, s, 0) \equiv 0$ and A is a constant matrix which every eigenvalues have negative real parts.

Theorem. *Assume the following conditions :*

(i) $||e^{At}|| \leq K_0 e^{-\alpha t},\ \alpha > 0,\ K_0 > 0,\ ||.||$ *is the usual norm in* R^n.

(ii) For any $L > 0$ *there exists* $\delta(L) > 0$ *such that for* $||u(t)|| \leq \delta(L)$ *there is valid*

$$||K(t,s,u_1) - K(t,s,u_2)|| \leq L||u_1 - u_2||,$$

for every $(t,s,u_1),\ (t,s,u_2) \in \Omega_1$,

$$||f(t,u_1,v_1) - f(t,u_2,v_2)|| \leq L\left[||u_1 - u_2|| + L\int_0^1 ||u_1(s) - u_2(s)||ds\right],$$

for every $(t,u_1,v_1),\ (t,u_2,v_2) \in \Omega_0$.

Then the trivial solution of (2) is stable for $|\lambda| < \frac{\alpha}{2L(1+L)K_0},\ t \in J$.

Acknowledgment. This work was supported by the plans of investigations MSM 262200013 and MSM 262200022 of Czech Republic.

DEPARTMENT OF MATHEMATICS, FACULTY OF ELECTRICAL ENGINEERING AND COMMUNICATION, BRNO UNIVERSITY OF TECHNOLOGY, TECHNICKÁ 8, 61600 BRNO, CZECH REPUBLIC
e-mail address: smarda@feec.vutbr.cz