REFLECTING POINT OF BISTATIC ALTIMETRY – GEOMETRIC MODEL WITH THREE QUADRICS

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Abstract

Good knowledge of Earth shape and its dimensions and field of gravity is a base for some kind of human activity. It is important for oceanography, climatology and similar sciences. Satellite altimetry is the way to obtain this information. And with bistatic altimetry we can obtain more information at the same moment.

This paper describes the ground of bistatic altimetry and the way to compute a reflecting point as an intersection of three quadrics. The first two quadrics are rotational ellipsoids and the third one is rotational cone. One global and two local coordinate systems are used in the solution. Transformation matrices are used for the change of coordinates between these coordinate systems.

In the first step of computing the reflecting point, we have to find the points of intersection between rotational cone and ellipsoid of reflecting points. In the second step, these points of intersection are tested if they belong to the reference ellipsoid WGS 84.

Keywords

Altimetry, Bistatic Altimetry, Rotational Quadrics, Intersection of Quadrics

1 The Ground of Bistatic Altimetry

Satellite altimetry is a method how to find out altitude of satellite with radar altimeter. It serves as an exact tool for finding out parameters of gravity field of the Earth and detailed model of the ocean geoid. Both can be used in some sciences, e.g. geodesy, oceanography and climatology.

In "classic" satellite altimetry, the same satellite is used as a sender and a receiver of radar signal. In bistatic altimetry, one satellite is used as a sender of radar signal and another satellite as a receiver of signal reflected by the earth surface (see Fig. 1).

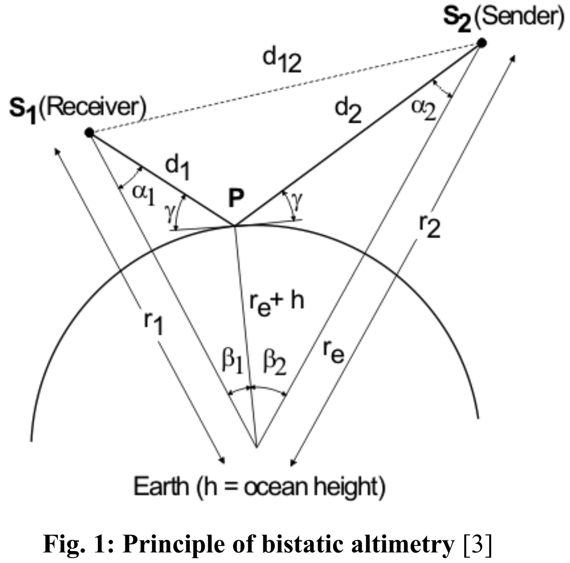

Fig. 1: Principle of bistatic altimetry [3]

GPS (Global Positioning System) satellites can be used as senders of the radar signal. These satellites gravitate at the high earth orbit, at the altitude of 20200 kilometers.

Satellites at low earth orbit (LEO), at the altitude of 300 to 470 kilometers, can operate as receivers, for example the CHAMP satellite (Challenging Minisatellite Payload for geophysical research and application).

Bistatic radar altimetry on the ocean surface with two satellites was presented in 1993 by Rubaškin. His model used one satellite at LEO as a sender and the other satellite at Geosynchronous Orbit as a receiver. In the same year, Martin Neira suggested and described a system of bistatic altimetry with GPS satellites [1].

In 2001, doc Klokočník and Carl Wagner were asked for assistance. They presented a simulation of the position and geometry of reflecting points on the ocean surface for a model with GPS and CHAMP [2]. They further worked on error analysis, measurement errors of time delay and orbital errors that affect the altitude of reflecting point on any surface.

None of the error models [2], [3] deal with the position of a reflecting point. Probably the first mathematical model for finding the reflecting point was presented in the paper [4].

The advantage of the concepts from papers [2] and [3] is the finest covering of the earth surface with reflecting points. Until this time, no significant measurement was carried out.

2 Task definitions

At the beginning, we know this data:

- position of GPS satellite
- position of CHAMP satellite
- velocity vector of CHAMP satellite
- signal length between GPS satellite and CHAMP satellite reflected by the earth surface
- angle between velocity vector of CHAMP satellite and direction of received signal

We have to compute the coordinates of the reflecting point on the earth surface.

3 Solution

According to the mathematic model proposed in the paper [4], the reflecting point is computed as an intersection of three quadrics. These quadrics are:

- the reference ellipsoid WGS 84
- the rotational ellipsoid defined by GPS satellite, CHAMP satellite and by the length of the reflected signal. The satellites determine the foci of this ellipsoid. The main half axis size can be derived from the signal length. The ellipsoid rotates about main half axis.
- the rotational cone defined by velocity vector of CHAMP satellite and by the angle between the velocity vector and the reflected signal. The velocity vector determines the rotational axis. The satellite is in the vertex of a cone.

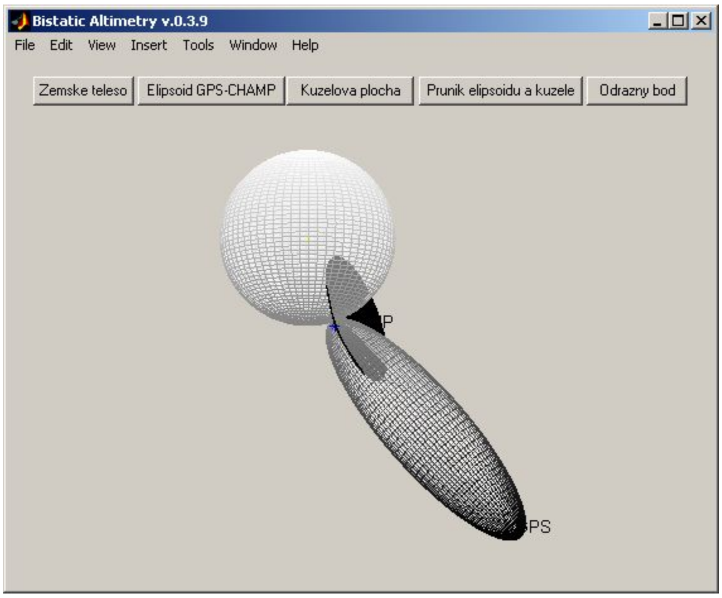

Fig. 2: Configuration of quadrics for GPS altimetry

At first, we compute the intersection of the ellipsoid of the reflecting points with a rotational cone. At random distance from the vertex of a cone, we select a parallel circle (reference circle). On the reference circle we choose dividing points. At the dividing point, we determine a ray that begins in the vertex of a cone and passes through this dividing point. Then we determine its intersection point with the ellipsoid of reflecting points.

In this calculation, we use two local coordinate systems. Both have a center in a point defined by coordinates of the CHAMP satellite. Axes of the coordinate system for ellipsoid of reflecting points (frame SSE) are oriented in the following way:

- axis x' is oriented in the direction of vector $GPS - CHAMP$
- axis y' is determined to be orthogonal to the axis x' and the velocity vector of the CHAMP satellite
- axis z' is computed so that basis set vectors are unit and determine the positive oriented coordinate system

Axes of the coordinate system for rotational cone (frame SSC) are oriented in the following way:

- axis x'' is oriented in the direction of velocity vector of the CHAMP satellite
- axis y'' is determined to be orthogonal to the vector $GPS - CHAMP$ and axis x''
- axis z'' is computed so that basis set vectors are unit and determine the positive oriented coordinate system

So defined coordinate systems have the advantage in common centre. They are turned through an angle about axis $y' = y''$ (see Fig. 3).

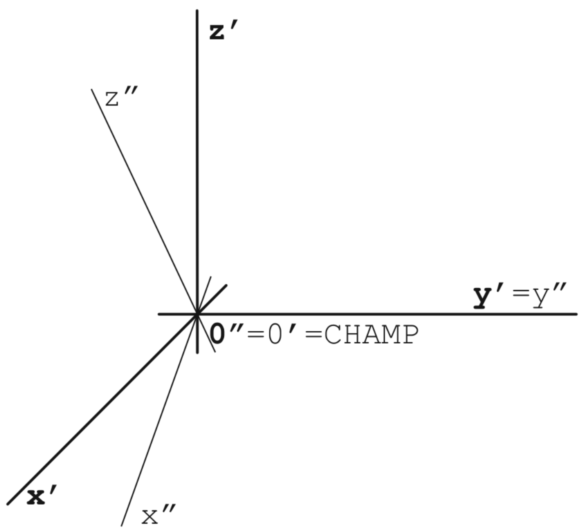

Fig. 3: Common position of local coordinate systems

Then for points of the cone it holds true

$$x'' = r$$
$$y'' = r \tan\omega \cos\varphi \,,$$
$$z'' = r \tan\omega \sin\varphi$$
(1)

where ω is the angle between the velocity vector of the CHAMP satellite and the reflected signal. The surface line of the cone, on which lays the selected dividing point K_i, is, after the change of coordinates from the frame SSC to the frame SSE, expressed as follows:

$$x' = t\,u$$
$$y' = t\,v \,,$$
$$z' = t\,w$$
(2)

where u, v, w are coordinates of the point K_i in the frame SSE.

After substituting the equation (2) into the equation of the ellipsoid of the reflecting points

$$\frac{(x' - x'_F)^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{b^2} = 1$$
(3)

we obtain a quadratic equation for parameter t. Its answer is

$$t_{1,2} = \frac{2ux'_F b^2 \pm \sqrt{(-2ux'_F b^2)^2 - 4((u^2b^2 + v^2b^2 + w^2a^2)(b^2 x'^2_F - a^2b^2))}}{2(u^2b^2 + v^2b^2 + w^2a^2)}$$
(4)

By inserting the positive value of parameter t into the equation (2), we obtain coordinates of the point L_i of the break-through curve of the ellipsoid and the cone (Fig. 4).

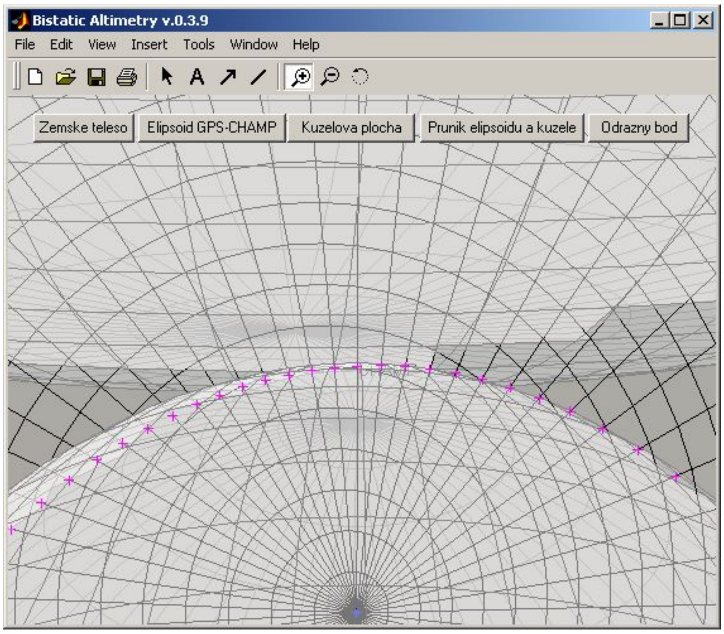

Fig. 4: Intersection of ellipsoid of reflecting points and cone

Now we test if some of the points L_i lie on the surface of the reference ellipsoid WGS 84.

If we used a constant step for choosing the dividing points of the reference circle, we would not be able to find the reflecting point. For that reason we sectionalize the reference circle to four parts ($\varphi = \pi/2$, $\varphi = \pi$, $\varphi = 3\pi/2$, $\varphi = 2\pi$). Then we allot the values of parameter φ for which the points of break-through curve are the closest to the reference ellipsoid WGS 84. Then we look for the reflecting point by cutting the interval of the parameter φ ($\varphi_{i+1} = \frac{\varphi_i + \varphi_{i-1}}{2}$). If the entered quadrics have no common point of intersection, the computation ends at the moment when the distance between the points on the break-through curve is less than 0,01 meters.

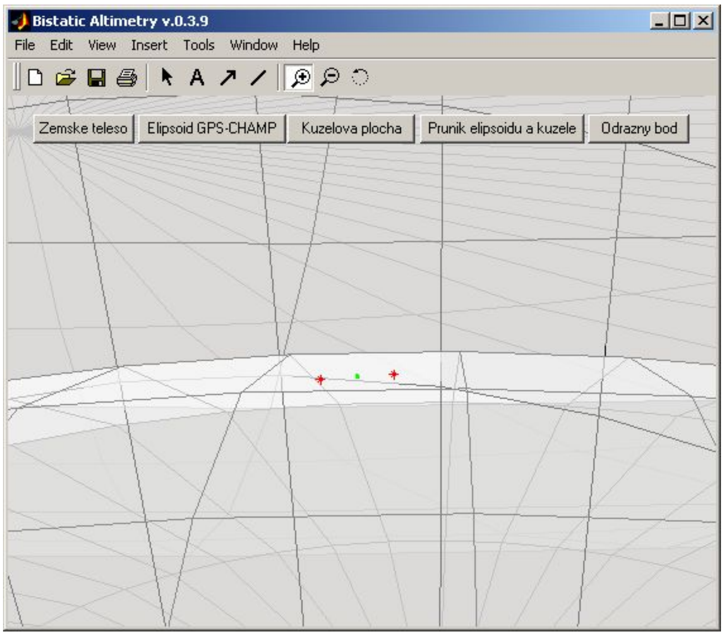

Fig. 5: Reflecting points of bistatic altimetry

On the Fig. 5, there are two reflecting points that have the following coordinates:

$$X_1 = 1750477{,}7 \qquad X_2 = 1752534{,}7$$
$$Y_1 = 1048874{,}0 \qquad Y_2 = 1026022{,}2$$
$$Z_1 = \text{-}6022001{,}7 \qquad Z_2 = \text{-}6025343{,}3$$

4 Summary

If we compute the reflecting point as an intersection point of two rotational ellipsoids and rotational cone in the configuration of bistatic altimetry, we can find two points, one point or no point.

In our case, we calculated two points of intersection, while the reflecting point has to be only one. If we use the law of reflection as an additional condition, there remains none of these reflecting points. A minor difference between angle of incidence and angle of reflection is $\Delta\alpha_2 = 1^\circ 21' 30''$. A major angle difference is $\Delta\alpha_1 = 2^\circ 20' 08''$. There remains a question, whether it is caused by rounding errors and error in the measurement of signal or the reflecting point is located somewhere else.

The signal sent out by the GPS satellite is reflected actually by the earth surface (geoid) and not by the reference ellipsoid.

Reference

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