History of the concept of vector space in university teaching of analytic geometry

Michal Novák

Abstract

Modern analytic geometry relies on the means of linear algebra. In the article I present a brief history of how this approach entered university curricula. I concentrate on Czech universities.

History of $n-$dimensional geometry[1]

The history of $n-$dimensional geometry can be traced as back as 14^{th} century to Nicole Oresme and the German and Italian medieval mathematicians. Yet the real beginnings of a concise theory of $n-$dimensional geometry first occured in the 1840s and 1850s.

Die lineale Ausdehnungslehre by Hermann Günther Grassmann published in 1844 first introduces what later became called the vector space theory. By defining "extendable images", "magnitudes" (of a given grade), and their systems Grassmann in fact proposed its main ideas. Showing properties of such systems he reached the ideas of linear dependance and independance and vectorial and mixed product. His theory was the first one to allow extensions to spaces of any dimension.

By the end of 19^{th} century the theory of vector spaces (and of $n-$dimensional geometry) was firmly established thanks to the second edition of Grassmann's work (*Die Ausdehnungslehre*, 1862) and the works of Ludwig Schläfli (*Theorie der vielfachen Kontinuität*, 1851 but published only as late as 1901) and Julius Plücker. Some progress was also achieved by Giuseppe Peano in his *Calcolo Geometrico*, 1888, but this work was not of great influence as it stood away from the mainstream.

Linear algebra in university analytic geometry

The ideas of Grassmann were not immediately followed. In fact, the roughness of his style and the abstract nature and unintuitiveness of his results prevented them from widespread adoption. Even though the second edition of *Die Ausdehnungslehre* marks a great progress in this area (as further works of Schläfli, Plücker and Peano do), the vector space theory did not enter university analytic geometry until well 20^{th} century. Let us follow its progress on the example of Czech universities and Czech university textbooks.

[1]For more details on this topic see e.g. [4] and (for G. Peano) [1].

First ocurrences of the term *vector* at Czech universities

The first to communicate Grassmann's ideas to Czech university students was most likely Matyáš Lerch, who had intensive contact with current mathematics abroad. Short after his habilitation at the Prague technical university (1886) he announced a 3–hour lecture *Analytic geometry of rational planar lines*[2] *(with a special regard to Grassmann's theory and to the theory of equipolence)*[3]. This was only a minor event as the lecture was only an optional one. Furthermore, in later years Lerch read only a 1–hour lecture *Analytic geometry of conic sections*, whose content, however, was different from the former lecture and did not include Grassmann's results.

The term *vector* does not appear in university textbooks of analytic geometry (and thus was probably not widely utilized during lectures) until 1919. The second edition of the popular Jan Vojtěch's textbook *Základy mathematiky ke studiu věd přírodních a technických*, which was a standard textbook at Czech technical universities, first comprises two rather short chapters "Vectors in plane" and "Vectors in space". Both of them, however, stand clearly apart from the text and have no connection with the reasoning on the topic.

The textbook gives the following definition of a vector:[4]

> *We call a magnitude of a certain size, certain direction and orientation a vector. In contrary to this a magnitude, the size of which is only important, is called a scalar. Some examples of vectors are segment of a line with a given direction and orientation, shifting, velocity, strength, etc. whereas a number, length of a line segment, time, mass, temperature, power, etc. are scalars.*

Even though some basic operations on vectors are mentioned (sum and decomposition), the concepts of vector space or linear dependance or independance are not studied. The purpose of these chapters is purely practical as they help in solving some practical tasks. Thus only the concepts of scalar, vectorial and mixed products are utilized.

Analytic geometry in the interwar period

The attitude towards linear algebra as a part of analytic geometry did not change in the interwar period. In Czechoslovakia (as well as abroad) geometry was still lectured in the traditional way in the interwar period. Even though general means used changed and reflected the overall progress in mathematics (homogeneous coordinates, projective geometry, differential geometry), analytic geometry – despite using all these means – still lacked the unifying concept of linear algebra.[5]

---

[2] In Czech *racionalných čar rovinných*, thus in fact of planar curves.

[3] The lecture was scheduled for the year 1887/8.

[4] [9], p. 16. The Czech original reads:

> *Veličina určité velikosti určitého směru a smyslu sluje vektor; proti tomu veličinu, u které jde pouze o velikost, nazýváme skalárem. Vektory jsou př. úsečka daného směru a smyslu, pošinutí, rychlost, síla, a pod., skaláry jsou číslo délka úsečky, čas, hmota, teplota, práce, a.j.*

[5] For more details refer to the university textbooks representative for this period: [2] and revised editions of [9].

Change in perception

The turmoiling times after 1945 saw many radical changes in perception of analytic geometry, mathematics and teaching in general. The demand of university education for teachers at schools of all grades[6] increased the number of university students. A number of school reforms took places in the late 1940s and 1950s.[7] One of their important outcomes was a creation of specialised teacher training university institutions[8] which used the latest teaching methods and techniques. This resulted in the change of perception of teaching analytic geometry which was now based on the concept of vector. It was at these new institutions that the change was introduced first. At universities with their firm establishement the changes were more gradual.

Such a radical change in the attitude towards the subject matter implies a slow and well staged transition. However, the first effort to present university students with the $n-$dimensional vector geometry was not a successful one. Despite being one of the first world's textbooks of its kind, *Základy analytické geometrie* by Eduard Čech was a book difficult to read. It completely abandoned the traditional approach to the issues of analytic geometry (conic sections and quadratics) and with only minor exceptions explains the subject matter in a space of general dimension. Due to its lack of illustrativeness it could be best referred to as a "dead born baby".

Čech's book could not be widely used during prospective teachers' training. Compared to *Základy analytické geometrie*, textbooks by Emil Mastný and Emil Kraemer look rather modest. They study only cases of $\mathbf{E_2}$ and $\mathbf{E_3}$ (*Analytická geometrie lineárních útvarů* by Kraemer only linear elements) but they do it in a slow, instructive and illustrative way. Another textbook published in the 1950s with a great influence on students of classical universities was a book by *Bohumil Bydžovský*, a leading figure of Czech university mathematics since short after World War I.

Teaching analytic geometry at technical universities changed as well. By the begining of 1950s only revised editions of Vojtěch's [9] textbook could serve as modern technical university teaching materials. *Analytická geometrie* by Jiří Klapka (1960) belonged to a set of new textbooks for technical universities published as a reaction on growing discrepancies between existing texts and demands of practice. It fully relies on the concept of vector and means of linear algebra. As a technical university textbook it – naturally – carries out its reasonings only in $\mathbf{E_2}$ and $\mathbf{E_3}$.

$n-$dimensional analytic geometry

Klapka's book marks a peak of analytic geometry at technical universities. A number of new subjects has been introduced since its publication and curricula have changed to include methods more convenient for technical practice. At universities, however, the situation has developed differently. In prospective teachers' training analytic geometry built on the means of linear algebra is a suitable tool for studying its classical subject matter – linear elements, conic sections

[6] Prior to 1946 only "gymnázium" teachers were required to have a university degree.

[7] Not only redefining the relationship of universities and the state. They were mostly organisational reforms creating new institutions, merging old ones and adjusting their roles.

[8] Faculties of education (at universities) and independant "Vysoké školy pedagogické".

and quadratics. Techniques resulting from this union may also be used in other parts of university mathematics.

The central theme of analytic geometry at universities in 1960s was strenghtening its algebraic part. In contrary to 1950s, university analytic geometry of 1960s focused on generalising ideas and expanding the vector space concepts to spaces of $n-$dimensions. We also finally see a clear cut distinction between projective, affine and Euclidean geometry.

Reference

[1] Bečvář, J.: Peanovo Calcolo Geometrico z roku 1888. Logika a teorie množin, geometrický počet a lineární algebra. In *Filozofické a vývojové problémy matematiky, Sborník vybraných přednášek z letních škol "Světonázorová výchova v matematice"*, 149–172 Praha: JČSMF 1988

[2] Bydžovský, B.: *Úvod do analytické geometrie.* Praha: Nakladatelství ČSAV 1956

[3] Čech, E.: *Základy analytické geometrie.* Praha: Přírodovědné nakladatelství 1951 (I. díl) a 1952 (II. díl)

[4] Čižmár, J.: Začiatky a formovanie základov $n-$rozmernej geometrie. In *Matematika v 19. století*, 57–81 Praha: 1996

[5] Klapka, J.: *Analytická geometrie.* Praha: SNTL 1960

[6] Kraemer, E.: *Analytická geometrie lineárních útvarů.* Praha: Nakladatelství ČAV 1954

[7] Mastný, E.: *Úvod do analytické geometrie lineárních útvarů a kuželoseček.* Praha: Nakladatelství ČSAV 1953

[8] Novák, M.: *Historie výuky analytické geometrie na vysokých školách v českých zemích (1848–1960).* Brno: 2002 (disertační práce)

[9] Vojtěch, J.: *Základy mathematiky ke studiu věd přírodních a technických.* Praha: JČMF 1919 (2. vyd.)

Mgr. Michal Novák, Ph.D.
Ústav matematiky
FEKT VUT v Brně
Technická 8
616 00 Brno
email: novakm@feec.vutbr.cz