Mathematical Methods in Geometric Modelling

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A short review of various modelling methods used in geometric modelling is presented in the paper. Methods of geometric modelling are used for construction of precise mathematical description of a real object or a simulation of some process. This construction is usually a set of specific operations performed by computer, which are necessary for creating a model of a real object, its storing in a database, analysis, modifications a visualisations. Methods of geometric modelling are formed as synthesis of principles and algorithms from various disciplines, parts of mathematics as e.g. analytic (coordinate) and descriptive geometry, topology, set theory, differential geometry, numerical analysis, linear and vector algebra, functional analysis, and others. There are described wire-frame models, polygonal schemes and boundary representation, parametric, variational and sweep models and free-form modelling principles, constructive solid modelling and fractal fuzzy modelling. Finally, free-form deformations of Bèzier solid cell basic grid and tetrahedral meshes are discussed., as the latest methods of geometric modelling of space figures

Grid of $(m + 1)$ x $(n + 1)$ x $(l + 1)$ points in the space

$$P_{ijk},\ \ 0 \le i \le m,\ 0 \le j \le n,\ 0 \le k \le l$$

defines the basic space, where a Bèzier solid cell is determined on the region $\Omega = [0,\ 1]^3$ by the differentiable point function of three variables

$$\mathbf{r}(u,v,w) =$$

$$= \sum_{i=0}^{m} \binom{m}{i} u^i (1-u)^{m-1} \sum_{j=0}^{n} \binom{n}{j} v^j (1-v)^{n-j} \sum_{k=0}^{l} \binom{l}{k} w^k (1-w)^{l-k} P_{ijk}$$

Interior curvilinear coordinate system of solid is (X_0, **U**, **V**, **W**).

Created deformed figure is inserted into the grid (Fig. 1), while curvilinear coordinates (u,v,w) of any point X of the figure can be determined in the form $X = P_{000} + u\mathbf{U} + v\mathbf{V} + w\mathbf{W}$.

Deformation of the space is defined by the change in the position of the vertices P_{ijk} in the basic grid and by creating a new grid determined from points Q_{ijk}, while curvilinear coordinates of point X_{def} after deformation are given by formula

$$X_{def} = r_{def}(u,v,w) =$$

$$= \sum_{i=0}^{m} \binom{m}{i} u^i (1-u)^{m-1} \sum_{j=0}^{n} \binom{n}{j} v^j (1-v)^{n-j} \sum_{k=0}^{l} \binom{l}{k} w^k (1-w)^{l-k} P_{ijk}$$

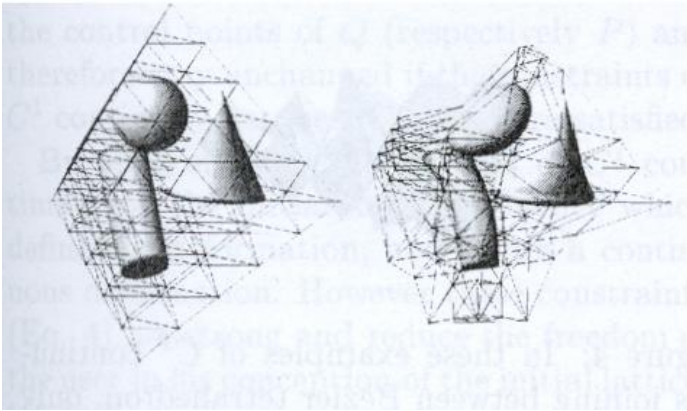

Figure 1. Free-form deformations of solids

Setting the initial conditions determining the type of the deformation (invariant some of the geometric characteristics - area, volume, boundary, and others.), specific deformations can be defined, e.g. interior deformation, volumetric, and others.

Analytic and synthetic representation of geometric figures are two possibilities of the formalised description of geometric figures, where the latter one is dominant with respect to the modelling process, for which geometry and geometric attributes of model have a prior meaning, not the mathematical formulas.