Heat transfer and air flow in buildings *

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The reliable prediction of development of temperature fields, driven by quasiperiodic climatic changes in outer environment, in building structures and constructions belong to most important aims of modelling in civil engineering. The most simple mathematical models are based on the classical one-dimensional Fourier - Kirchhoff differential equation of heat conduction (of evolution type, in general) with an unique unknown scalar field of temperature, supplied by a boundary condition of heat convection. This approach needs only some elementary knowledge from the theory of ordinary differential equations; moreover, at least in the stationary case with constant material characteristics the formal solution is well-known and compatible with some practical observations and laboratory measurements. The one-dimensional formulation can be extended to the two- or three-dimensional one and transformed to a weak form, containing corresponding boundary conditions in the natural way.

Unfortunately, the above sketched approach gives very bad results in the complex analysis of buildings as thermal systems with a lot of rooms, designed for human activities. On the other side, the thermal processes in such buildings (by the Czech technical standards) must be evaluated to verify criteria of two types: i) of thermal stability (certain lower and upper bounds for the temperature in rooms are prescribed, also the rate of temperature changes has to be slow enough, nearly independent of outer climatic changes), ii) of energy cost (heating and air-conditioning is not allowed to be expensive). The crucial difficulty in all calculations is that (inside the Fourier - Kirchhoff theory) we do not know how to include thermal processes in large empty rooms (containing only air) into our simplified model. One possible and frequently used (non-physical) trick is to assume that the temperature in every point of a separate room is the same, variable only in time; the temperature in walls, roofs, etc., is evaluated using the Fourier - Kirchhoff approach.

Nevertheless, physically correct formulations should respect (at least approximately) the basic conservation principles – of mass, momentum and energy. There exist two important ways for heat propagation in time: i) the classical conduction (as discussed above) and ii) the transfer caused by air flow in rooms. From the analysis of physical properties of air we can expect that the process ii) will be dominant and the process i) (unlike solid structures) nearly negligible; the main reason of this fact is that the factor of heat conduction of air is very low. In general, the conservation of mass gives one continuity equations, the conservation of inertia gives three equations of viscous air flow of and the conservation of energy gives one equation of heat transfer which can be identified with certain generalization of the Fourier - Kirchhoff equation. These equations involve four fields of unknown quantities of air – the temperature, the pressure, the flow rate and the density. We have only three types of differential equations; the last condition is the simple

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algebraic relation between the temperature, the pressure and the density, popular as the Gay - Lussac law. For the flow rate only homogeneous Dirichlet boundary conditions can be considered.

The weak (integral) formulation connects one equation of heat transfer with boundary conditions of heat convection and involves above mentioned Dirichlet boundary conditions into three equations of air flow. The pressure can be calculated directly using the remaining unknown quantities; therefore one continuity equations completes the system. Unfortunately, the mathematical theory of such evolution systems with some admissible initial conditions includes many open questions; their numerical analysis is difficult and expensive. Thus, our aim is to suggest some simplifications, neglecting non-substantial phenomena, to derive a (relatively simple) numerical algorithm, implementable on standard personal computers without special hardware or software requirements. Such algorithm should be able to be incorporated into more complex software systems for the thermal analysis of structures, too. The derivation of such algorithm is not trivial: namely the equation of air flow is strongly nonlinear, the viscosity term is very low, but cannot be removed without risk of the loss of solvability.

We can expect the flow with a low Mach number; thus we can treat the density as constant in the unsteady and convection terms and variable only in the gravitational term (this is the so-called Boussinesq approximation). The assumption of isothermal changes enables us to formulate a simple relation between the density change and temperature change (using the coefficient of volumetric expansion). Consequently (using one additional formal transformation) the pressure can be removed completely; in this way we obtain only three equations of air flow and one one equation of heat transfer; moreover, the choice of the air flow rate must preserve an additional condition.

The MATLAB-based original software code has been developed for the study of problems of this type. For simplicity only a vertical cut of a (sufficiently long) room has been considered. The standard implementation of MATLAB does not include any special means for the analysis of equations of the Navier - Stokes type (which is the most complicated here), but some functions from the toolbox PDE are useful (more advanced means, offered by FEMLAB, have not been tested yet). The method of discretization in time (based on the construction of Rothe sequences of approximate solutions in discrete time levels) has been applied together with the finite element method. The large systems of nonlinear algebraic equations are solved with help of the mathematical library NAG; the SIMPLE algorithm, controlling an additional condition for the air flow rate, makes use of MATLAB-supported matrix pseudoinversions.

The extended CD-version of this contribution (13 pages) includes the overview of applied equations together with useful references (for their derivation and transformation) and the more detailed software description (it covers also heat transfer in porous layered walls where other serious difficulties occur – namely the moisture-determined change of material properties). Moreover, a typical example of the time-dependent redistribution of temperature in a room of size (in the vertical cut) $5\,\mathrm{m} \times 5\,\mathrm{m}$, caused by the relatively quick temperature growth from 19°C to 21°C on an outer wall, with the dominant air flow is presented.