Mathematical Modeling and Informatics in Electrical Engineering

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1. Mathematical modeling

The paper analyses the methods of numerical solution of various tasks defined in terms of boundary-initial value problems of differential or integral equations. In natural sciences and engineering, a mathematical model is generally defined by differential equations which can be solved using mostly:

- finite-difference method (FDM),
- finite-element method (FEM),
- boundary-element method (BEM).

The choice of calculation method is affected by many factors. First of all, it is determined by the possibility of an accurate definition of the problem and regard to the boundary-initial conditions. It also depends on the system of algebraic equations and parameters of computer hardware used for carrying out the task.

The FDM is the oldest, simplest, and most popular method, based on substituting finite differences for derivatives. It was formulated as an approximate discrete method for solving boundary-value problems defined in terms of differential equations. Furthermore, its application was extended to variational problems. It may also be applied to boundary-initial value problems related to differential equations of parabolic type, describing a temperature distribution in the theory of heat conduction [7]. Nevertheless, for the domains of complex geometry, it becomes too much complicated due, first of all, to boundary conditions. These difficulties may be avoided if the calculation is made by FEM.

The last method is applied mostly to elliptic equations. However, when difference discretization with respect to time is introduced, it may also be used for parabolic equations. An equation is discretized by dividing the examined field into separate triangles. i.e. by triangularizing it. Afterwards, the base functions are formulated. FEM requires an explicit definition of the field subject to analysis. Therefore, it is usually applied to internal problems. It might also be used for external problems but leads to huge systems of equations, as the elements must cover the whole field of analysis.

Recently a BEM approach has very often been used for mechanical problems. It helps to reduce the number of equations considerably. It appears to be very advantageous, the solving of a large number of equations requires an immense memory size and is rather time consuming. The success of the method is based on omitting the discretization of the field. Only its border is subject to discretization, causing a reduction of the size and cutting down the computation time. The FEM and BEM approaches might be considered as complementary as their faults and advantages compensate each other. Extensive work aimed to connect both methods lead to hybrid techniques combining advantages of both and eliminating their faults .

Unfortunately, the above-mentioned methods are of little avail in determining the voltage gradient distribution in electro-insulation systems. The integral equation method (IEM) seems to be the most appropriate for this purpose, as electric potential distribution is described with the help of integral equations [2]. Such formulation of the problem, including boundary conditions (potentials for the conductors or their total charges) leads to a system of integral equations the numerical solution of which makes it possible to determine the charge density distribution of conducting parts and potential distribution in the surrounding space . In general, IEM is directly conducive to a mathematical model described by various classes of integral equations or, indirectly, to a boundary-initial problem of some differential equations. The method is applied to the theory of heat conduction [7] and to the theory of diffusion. It is also used in electromagnetism, for example in determining selected electrodynamical parameters in three-phase systems of shielded heavy-current busways . The choice of the method in such a case is justified, as the distribution of current density in phase conductors is obtained by solving a system of integral equations. The current density distribution of a single live conductor described in [6, 8] was found with the use of geometry of the system. The kernel of the equation was formulated with the account of skin effect of phase conductors, their approaching, and inducing of eddie currents in the shield. By such an approach, solutions were obtained that could be used to analyze electromagnetic phenomena occurring in current busways and the neighboring space, confining the considerations to a particular part of the field under investigation, i.e. to the surface area of one of the live conductors. Attempts at integrating electromagnetic field equations have a long history which is documented by a large number of publications [2, 6, 8]. The integral representation of field equations serves as a basis for many analyses utilizing numerical methods, particularly in unrestrained regions.

The aim of this paper is to present the advantages of the method of integral equations (IEM) and the possibilities of its application to various branches of engineering, particularly to the problems arising in power engineering [2]. It is an analytical-numerical method and requires great efforts from highly skilled specialists (mathematicians, computer scientists, engineers). IEM seems to be a natural method, especially in the field of electrodynamics. This is a case of electromagnetic field being described with integral equations, the kernels of which are searched for by integral transformations in the domain of space variables, while in the time domain the expected system response has a form of integral formulas. Moreover, an integral description of electromagnetic field in an environment inhomogeneous with regard to electric and magnetic features is presented in [8]. It confines the expected solution valid for the whole domain to a predefined part of the space subject to analysis. This enables a considerable reduction of the size of the system of equations. The method is worth of being applied as for the case of complex geometry of the problem, which is frequently met. In such a case, the minimization of the size of the systems of equations and the reduction of computation time, while maintaining the accuracy, becomes highly important.

Integral equations, or rather their systems, are often matched with mathematical models describing the current density distribution at the cross-section of a working conductor [6,8] or in the cartridge of an induction heater. The knowledge of the current density distribution may be a basis for determining some electrodynamic values such as magnetic induction or distribution of electrodynamic forces acting at selected points of the conductors.

Similarly, Maxwell's equations describing the relationship between the vectors of electromagnetic fields provide a system of partial differential equations that may be converted into integral equations.

Next considerations we restrict to the following integral equations of the mixed type

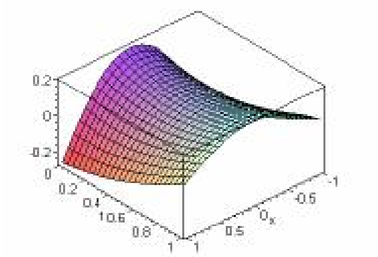
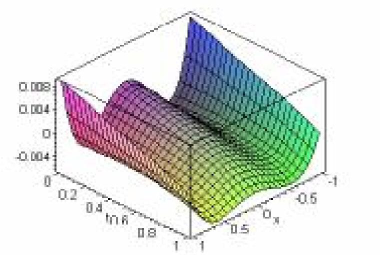
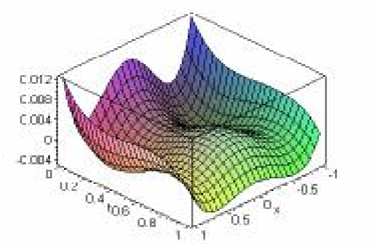
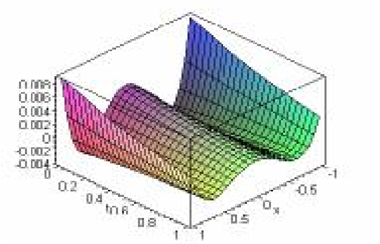
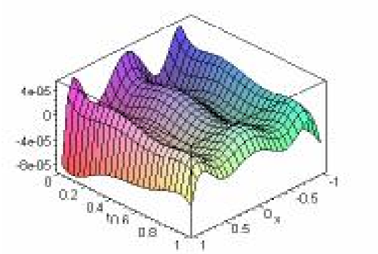
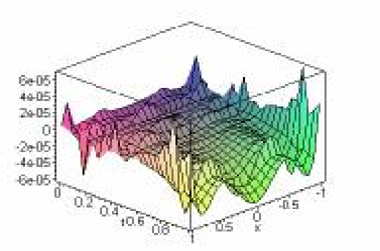
$$f(x,t) = g(x,t) + \int_0^t \int_M k(x,t,y,s) f(y,s) dy ds,$$

which generalize Volterra and Fredholm integral equations. The presented equations play a very important role in epidemiology, mechanics, electromagnetics and engineering. These equations arise in the heat conduction theory and the mathematical modelling of the spatio-temporal development of an epidemic. The spread of the disease in a given population can be described by the following mixed integral equations. Some initial-boundary problems for a number differential partial equations in physics are reducible to the above integral equation. Consider this equation in space-time, where g is a given function in the domain $D = M \times [0,T]$ (M is a compact subset of m-dimensional Euclidean space) and u is an unknown function in D. The kernel k is defined in the domain $\Omega = \{(x,t,y,s): x, y \in M, 0 \le s \le t \le T\}$.

The theory and computational methods were presented in [1]. In the next section, we will present a test-example for projection methods presented in [1].

2. Numerical experiments

$$f(x, t) = -\frac{\cos(x)\,(-6\,\mathbf{e}^{(-t)} - \mathbf{e}^{(-t)}\sin(2) + 2\,\mathbf{e}^{t} + \mathbf{e}^{t}\sin(2))}{4} + \int_0^t \int_{-1}^1 \cos(x-y)\,\mathbf{e}^{(t-s)}\,f(y, s)\,dy\,ds\,.$$

„GalClas_G”	„Gal_Four”
absolute error for n=2, m=2, k=3, total number of operations: **6180**	absolute error for n=3, m=16, total number of operations: **5526**
	
absolute error for n=3, m=3, k=3, total number of operations: **58320**	absolute error for n=3, m=61, total number of operations: **58041**
	
absolute error for n=5, m=8, k=3, total number of operations: **2521750**	absolute error for n=7, m=190, total number of operations: **2819285**
	

Graphs of absolute errors for „GalClas_G” and „Gal_Four” algorithms.

3. Conclusions

The aim of this paper is to show the advantage of the method of integral equations in electrical engineering.

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