REMARK ON POSITIVE SOLUTIONS OF RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

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In presented investigation conditions for the existence of positive solutions for systems of retarded functional differential equations (RFDE's) with *unbounded delay* and with *finite memory* are derived. A function $p \in C[\mathbb{R} \times [-1, 0], \mathbb{R}]$ is called a p -*function* if it has the following properties [7, p. 8]:

(i) $p(t, 0) = t$.

(ii) $p(t, -1)$ is a nondecreasing function of t.

(iii) there exists a $\sigma \geq -\infty$ such that $p(t, \vartheta)$ is an increasing function for ϑ for each $t \in (\sigma, \infty)$.

In the theory of RFDE's the symbol y_t, which expresses *"taking into account"*, the history of the process $y(t)$ considered, is used. With the aid of p- functions the symbol y_t is defined as follows:

Definition 1 *Let $t_0 \in \mathbb{R}$, $A > 0$ and $y \in C([p(t_0, -1), t_0 + A), \mathbb{R}^n)$. For any $t \in [t_0, t_0 + A)$, we define*

$$y_t(\vartheta) := y(p(t, \vartheta)), \ -1 \leq \vartheta \leq 0$$

and write

$$y_t \in \mathcal{C} := C[[-1, 0], \mathbb{R}^n].$$

We investigate existence of positive solutions of the system

$$\dot{y}(t) = f(t, y_t) \tag{1}$$

where $f \in C([t_0, t_0 + A) \times \mathcal{C}, \mathbb{R}^n)$, $A > 0$, and y_t is defined in accordance with Definition 1. This system is called the system of p-*type retarded functional differential equations* (p-RFDE's) or a system *with unbounded delay with finite memory.*
Positive solutions in the critical case were studied e.g. in [3]. Close results are contained e.g. in [1, 5]–[7]. Some known scalar results concerning existence of positive solutions were extended for nonlinear systems of RFDE's with bounded retardation in [2] and for nonlinear systems of RFDE's with unbounded delay and with finite

memory in [4]. Proofs of corresponding results, presented in enlarged version, are based on the retract method and the Lyapunoff method. These techniques were used e.g. in [4].

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