

Investigation of asymptotic behavior of solutions of discrete and differential equations

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A survey of author's latest results concerning asymptotic behavior of solutions of discrete equations achieved (together with his collaborators) by using a retract idea will be presented.

A powerful tool for investigation of various asymptotic, boundary-value and qualitative problems in the theory of ordinary differential equations as well as in the theory of delayed differential equations is the retraction method because retract principles very often play crucial role in the proofs of various statements. The utilization of retract idea in theory of ordinary differential equations goes back to T. Ważewski (e.g. Sur un principe topologique de l'examen de l'allure asymptotique des intégrales des équations différentielles ordinaires, *Ann. Soc. Polon. Math.* **20** (1947), 279–313; Ch. Conley, *Isolated Invariant Sets and the Morse Index, Regional Conference Series in Mathematics*, No 38, Providence R. I., American Mathematical Society, 89 p., 1978).

Recent extensions of the retract principle in the theory of discrete equations and corresponding applications will be a topic of contribution. A general principle giving a guarantee that the graph of at least one solution of systems discrete equations stays in a prescribed domain will be discussed. As an application sufficient conditions of existence so-called δ -bounded solutions will be derived.

An extension of mentioned general principle to discrete delayed equations will be developed.

A special attention will be devoted to existence of positive solutions. Application of general principle to the problem of existence of positive solutions of delayed discrete equations and new sufficient conditions of positivity will be presented as well. The phenomenon of existence of a positive solution of differential or difference equations often arises when we analyse mathematical models describing various processes. It is an opposite case to the phenomenon of oscillation of all solutions.

The existence of positive solutions is very often substantial for a concrete considered model. In biology e.g. when a model of population dynamics is de-

scribed by an equation, the positivity of a solution may mean that a concrete biological species can exist in the supposed environment. This is a motivation for intensive study of conditions of existence of positive solutions of differential and difference equations, as well as their properties. In this contribution conditions of existence of a positive solution are given for a general class of nonlinear delayed discrete equations. Results obtained indicate sufficient conditions for the existence of a positive solution and also give upper estimate for it. We apply general results to one class of concrete difference equations with delay and compare asymptotic behavior of solution with asymptotic behavior of solutions of corresponding differential equations with delay. With respect to the results in the existing literature we remark that only concrete classes of linear and nonlinear discrete equations were considered.